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A new approach to determine occupational accident dynamics by using ordinary differential equations based on SIR model

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The motivation of this study is to develop and establish an occupational accident dynamical model (OA model) based on Susceptible-Infected-Recovered framework. In order to investigate the dynamics of the OA model, monthly occupational accident data from Turkey between 2013 and 2020 has been selected as dataset. The OA model is defined by a coupled first-order ordinary nonlinear differential equation with four variables. In addition, the relationships between these variables are described with ten parameters. The OA model's characterization of the equilibrium points is analyzed by investigating the behaviors of these points according to the eigenvalues derived from the Jacobian matrix. Also, the stability of these points is obtained according to the eigenvalues. These results show the behavior of the system near equilibrium points. After that, the reproduction number is computed by using the next-generation matrix method. The calculated reproduction number for given parameters reveals that the OA model is unstable. The OA model is numerically solved in 96 steps, with a time interval of 1 month, using the ODE45 Matlab routine based on the explicit Runge–Kutta algorithm. In addition, a modified OA model is developed by adding the Occupational Health and Safety (OHS) re-training parameter to the OA model to observe a reducing effect on occupational accident numbers. The main results of this study provide a new approach about the future estimation of the number of occupational accidents. Furthermore, through the comparison of numerical results from both models, the study demonstrates that national safety policies, particularly those enhancing the efficacy of OHS training, can effectively mitigate accidents.

Keywords Occupational health and safety, Occupational accident, Nonlinear dynamics, Mathematical modelling

Occupational Health and Safety (OHS) is aimed at eliminating work-related injuries, diseases, and deaths[1]. The importance of this field is increased with the industrial developments and the technology in the world[2]. The matter of OHS benefits from engineering, ergonomics, physics, biology, law, medicine, and social science disciplines. Understanding and prediction of accident causation is the harmonic implication of these disciplines[3]. Occupational accidents originate in working area safety and require a proactive approach to identify hazards, minimize risks, and improve preventive measures[4]. Statistical data on occupational accidents provide valuable insights into workplace injuries' frequency, character, and results. These statistics often include many different kinds of industries, occupations, and regions, helping stakeholders (employees, employers, customers, visitors, and subcontractors, etc.), policymakers, and researchers gain an extensive understanding of the present state of occupational health and safety[5,6]. Analyzing these statistics enables the identification of hazardous areas and risk factors, facilitating targeted intervention and prevention policies aimed at reducing the occurrence of accidents[7]. In this context, Carnero and Pedgeral aimed to predict the future of work accidents as a trend forecast in Spain at different levels using multivariate components. Their data consists on occupational accidents that occurred between 1998 and 2009, published by the Spanish state[8]. In another study, Cheng et. al. investigated the characteristic factors of occupational accident data in small construction enterprises in Taiwan by using

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descriptive statistical methodologies and correlation coefficient analysis. The data used in their study were collected from the Taiwan Central Government Labor Affairs Council between 2000 and 2007[9]. Shafique and Rafiq presented the current occupational injury trend among Hong Kong construction workers. In their study, a statistical analysis of incidence and mortality rates was performed to evaluate each type of accident by evaluating the annual reports of the Hong Kong labor department between 2008 and 2018[10]. In a different study, Zhu et al. analyzed occupational accident data to evaluate the Occupational Health and Safety (OHS) policies performance within the coal mine industries. The study utilized the coal mine accidents that occurred between 2008 and 2021, as reported by the Ministry of Emergency Management of the People's Republic of China and the Nation Mine Safety Administration[11]. Kandemir and ¸Sim¸sek aimed at the costs and psycho-social effects of occupational accidents with a statistical study that relied on the data published by the Social Security Institution of Turkey. Unlike previous studies, the relationship between the gender of the accident victims, the working environment, the duration of incapacity for work, and the type of work accident was evaluated by using the log-linear analysis method and two different models based on their research. The amount of accidental cases is associated with the development level of countries[12]. Macedo and Silva examined officially recorded accidents and labor-related working conditions in workplaces between 1992 and 2001 based on their research. They stated that occupational accidents have a great impact on human life and also bring high costs for a country's social security system[13]. Occupational accidents are a significant problem that may disrupt human body integrity. It may result not only in death but also in serious injury and loss of limb. It is clear that occupational accidents have negatively affected the mental and physical health of society.

Complex systems, spanning disciplines such as physics, biology, ecology, physiology, sociology, and economy, often rely on nonlinear dynamic theory. These fields leverage numerical simulations to unravel intricate patterns, predict emergent phenomena, and understand the underlying mechanisms within these complex systems. Population dynamics is one of the most common areas of study for researchers due to its rich nonlinear interactions[14–17]. The first study of population dynamics, called the Malthusian growth model, was formulated by Malthus[18]. Then, the logistic equation model was introduced in 1845 to model the evolution of any population over time[19]. In this context, Lotka–Voltera equations, which are leaned on the logistic equation, are also used to describe ecological prey and predator models[20]. It has been shown that these equations are continuous in time evolution and show chaotic behavior when the degrees of freedom are three or more[21,22]. The spread of infectious diseases, like population dynamics can be described by nonlinear dynamical methods. This process may be carried out mathematically with the SIR, SEIR, SIRD, and SIVR (Susceptible-Exposed-Infected-Recovery-Death-Vaccinated) models depending on the investigated variables[23]. There are many studies conducted for this purpose. For instance, a SEIR model was developed for tuberculosis transmission (TB) with the time-dependent parameters[24].In another study, a predictor-corrector type nonstandard finite difference scheme on the SEIR model was implemented for measles transmission disease[25,26]. Modeling of COVID-19 infection was applied to different types of models such as SIR, SIRD and SIVR by considering many effects of crowding, transportation, and vaccination[27–29]. The HIV/AIDS transmission dynamic was explored based on the SIR model by using Runge–Kutta and Galerkin–Petrov time discretization of a high-order scheme[30,31]. In addition, the cause of ulcer and environment for the spread of Buruli disease was investigated by using a modified SIR model[32]. Although there are some studies about modeling and prediction of occupational accidents, as mentioned, a study on the non-linear dynamic modeling approach to occupational accident data has not yet been addressed in the literature. In this study, a new model has been proposed to describe the dynamics of occupational accidents. This model relies on population dynamic theory and the SIR model. The model has been solved numerically by the Runge–Kutta algorithm. The parameters' values have been found by fitting the real data. The error between the real and simulated data has been calculated by using normalized mean square error (NMSE). The stability analysis has been done, and finally, the effect of a new parameter, which refers to OHS re-training, has been investigated. This paper contains three main sections: First, the description of the Occupational Accident model (OA model) is presented under the Methods title. Following that, the stability analysis of the OA model is explained. Subsequently, the numerical results of OA and the modified OA model are examined. Finally, the contribution of the study is mentioned in the Conclusion section.

Methods

Description of OA model

In this section, a mathematical model based on the SIR model is implemented to illustrate occupational accidents. Therefore, the population is categorized into four variables: Susceptible (S), referring to the number of insured persons without accidents; Infected (I), representing the number of insured persons who have had accidents reported within 3 days (Minor accidents) and those with reports exceeding 3 days (Major accidents); and finally, Recovery or Death (R or D), representing the number of insured persons who have had fatal accidents. A diagram is shown in Fig. 1 to visualize the relationship between variables in the time-dependent OA model. In the diagram, diamond, circle, square, and triangle shapes represent a variable which are used in the model. The parameters defining the relations between the variables are illustrated by arrows. λ indicates the recruitment to the population. X population turns into A and B population. A transforms into B. Finally, B transform into Z. The arrowhead toward the variables is meant positive contribution. Similarly, the opposite side of the arrowhead indicates a negative contribution.

The occupational accident dynamics is investigated by constructing time-dependent first-order coupled ordinary differential equations. The equations are composed thanks to the diagram introduced in Fig. 1. The equation system of the OA model is given below (Eq. 1)[33,34],

$$\frac{dX}{dt} = \lambda N_1 + r_1 A + r_2 B - \alpha_1 X A - \alpha_2 X B - \mu X \tag{1a}$$

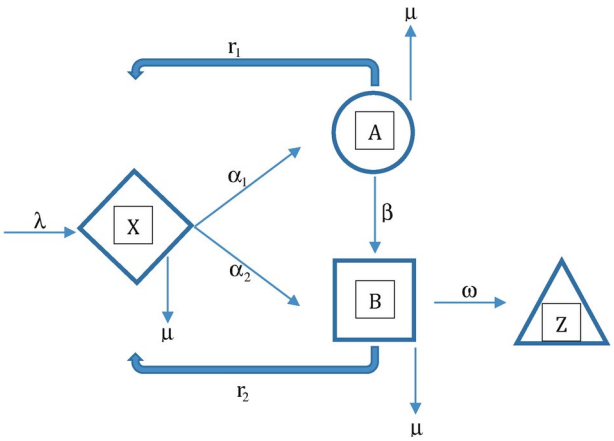


Fig. 1. Visual representation of occupational accident (OA) model structure.

Variables	Definitions
X	The number of insured without accident
A	The number of insured persons who had accidents reported within three days or less (Minor accidents)
B	The number of insured persons who had accidents reported more than 3 days (Major accidents)
Z	The number of insured persons who had fatal accidents
Parameters	**Definitions**
λ	Employment rate
α_1	Minor accident incidence rate
α_2	Major accident incidence rate
r_1	Minor accident recovery rate
r_2	Major accident recovery rate
β	Ratio of conversion minor accident to major accident
ω	Mortality rate of major accident
μ	Natural death rate
N	Average of compulsory insured number
N_1	Average of insured without accident number

Table 1. The definitions of variables and parameters.

$$\frac{dA}{dt} = \alpha_1 XA - (\mu + r_1 + \beta)A \tag{1b}$$

$$\frac{dB}{dt} = \alpha_2 XB + \beta A - (\mu + r_2 + \omega)B \tag{1c}$$

$$\frac{dZ}{dt} = \omega B \tag{1d}$$

where $X(t)$, $A(t)$, $B(t)$, and $Z(t)$ refer to the number of insured without accident, the number of insured persons who had accidents reported within three days or less (Minor accidents), the number of insured persons who had accidents reported more than 3 days (Major accidents), and the number of insured persons who had fatal accidents, respectively. The parameters λ, α_1, α_2, β, r_1, r_2, ω, μ, N, N_1 indicate employment rate, minor accident incidence rate, major accident incidence rate, ratio of conversion minor to major accident, minor accident recovery rate, major accident recovery rate, mortality rate of major accident, average of compulsory insured number, and average of insured without accident number, respectively. For the occupational aspects, minor and major accident has different importance. Thus, they and parameters related to minor and major accidents (α_1,α_2 ,r_1, and r_2) are defined specifically in OA model so that the effect of minor and major accidents can be seen. All the variables and parameters are summarized in Table 1.

In Turkey, the data regarding the number of people having accidents and insured are declared by the Social Security Institution (SSI) every year monthly[35]. In this study, the announced data of occupational accidents and insured numbers by SSI between 2013 and 2020 are selected. The OHS law in Turkey entered into force in 2013. Since employers started to use online systems to report work accidents,the occupational accident data has been

reliable since 2013. Therefore, the year 2013 is chosen as the beginning and the year 2020 is chosen as the ending period of the data (last announced by SSI).

The average number of compulsory insured N is selected as the universe of the system. X data is calculated by subtracting the number of accidents from N cumulatively. The universe of the system is defined $N(t) = X(t) + A(t) + B(t) + Z(t)$. In addition, A, B, and Z data are calculated cumulatively and scaled 1/1000000. Occupational accidents and insured data between 2013 and 2021 for Turkey are shown in Fig. 2.

While λ and μ parameters are declared annually by Turkish Statistical Institute (TUIK)[36], α_1 and α_2 parameters are calculated by SSI's annual occupational accident incidence rate (IR) given in Eq. 2 below,

$$IR = \frac{NEI}{(NDPA * 8) * 225000}, \tag{2}$$

where *NEI* represents the number of occupational accidents, and *NDPA* is the total working days of all insured persons during a calendar year. It is assumed that 100 insured people work 45 h per week, 50 weeks in a year, which means 225,000. Also, 8 is described as average daily working hours.

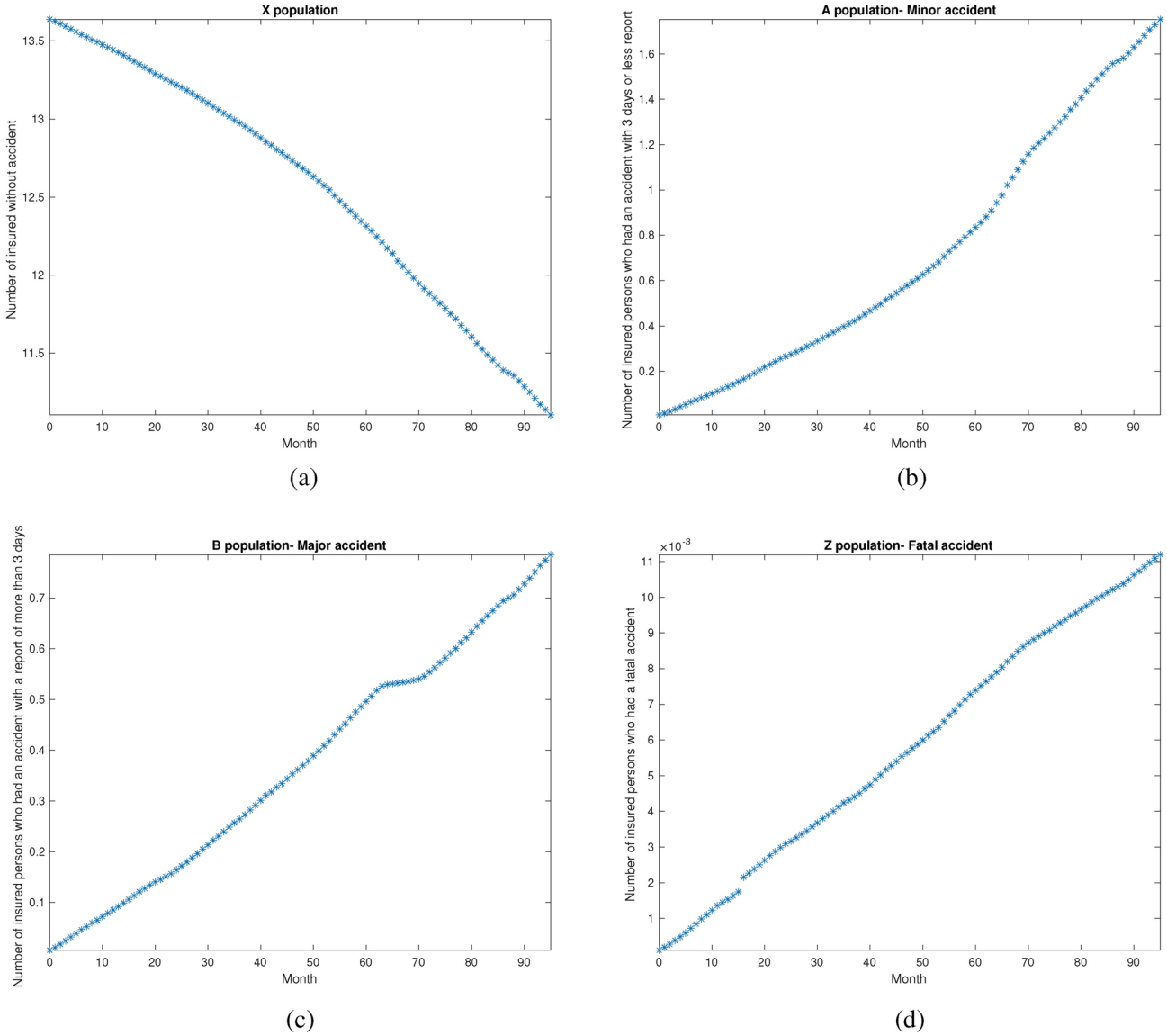


Fig. 2. Occupational accident and insured data in Turkey (between 2013 and 2021), (**a**) Number of insured without accident (Population X), (**b**) Number of insured persons who had accidents reported within 3 days or less (Population A-Minor accident), (**c**) Number of insured persons who had reported more than 3 days (Population B-Major accident), (**d**) Number of insured persons who had a fatal accident (Population Z-Fatal accident).

The parameters λ, μ, α_1, and α_2 are the average of the annual values announced between 2013 and 2020. However, β, r_1, r_2, and ω are found by using fminsearch subroutine in Matlab due to the lack of the parameters which are unpublished by the government. Initial value of β and ω are calculated 0.009 and 0.04 by using Heinrich's accident pyramid rate (300-accidents with no injuries, 29-accidents with minor injuries, 1-major injuries)[37]. Furthermore, r_1 and r_2 are selected arbitrary 0.001 and 0.01, respectively. fminsearch iterates these parameters and solves the equation numerically for each of them. Eventually, It finds optimum parameters' values by minimizing error between real and simulated data[38]. All parameters are scaled 1/1000000 and given in Table 2.

Stability analysis of OA model

In the model, the derivative of Z only depends on variable B. Since the derivative of Z does not affect the generality, it can be neglected. The vector field of the OA model is written below,

$$\begin{aligned}\mathbf{f} = (&\lambda N_1 + r_1 A + r_2 B - \alpha_1 X A - \alpha_2 X B - \mu X, \alpha_1 X A - (\mu + r_1 + \beta)A, \\ &\alpha_2 X B + \beta A - (\mu + r_2 + \omega)B)\end{aligned} \tag{3}$$

The divergence of the vector field can be used for classification stability. $\nabla.\mathbf{f} = (-\alpha_1 A - \alpha_2 B - \mu + \alpha_1 X - (\mu + r_1 + \beta) + \alpha_2 X - (\mu + r_2 + \omega))$. The divergence of the vector field OA model is always positive depending on parameters and variables values. Therefore, the model is non-conservative. The equilibrium points for the OA model can be calculated by using $f(x) = 0$ where $\dot{x} = f(x)$. The equilibrium points depending on the parameters are as follows in Eq. 4,

$$\begin{aligned} X &\to \frac{N_1 \lambda}{\mu}, \\ A &\to 0, \\ B &\to 0, \end{aligned} \tag{4a}$$

$$\begin{aligned} X &\to \frac{r_2 \omega + \mu}{\alpha_2}, \\ A &\to 0, \\ B &\to \frac{N_1 \alpha_2 \lambda - r_2 \mu - \omega \mu - \mu^2}{\alpha_2 (\omega + \mu)} \end{aligned} \tag{4b}$$

$$\begin{aligned} X &\to \frac{r_1 + \beta + \mu}{\alpha_1}, \\ A &\to \frac{(r_2 \alpha_2 + \omega \alpha_1 - r_1 \alpha_2 - \alpha_2 \beta + \alpha_1 \mu - \alpha_2 \mu)(N_1 \alpha_1 \lambda - r_1 \mu - \beta \mu - \mu^2)}{\alpha_1 (\omega \alpha_1 \beta + r_2 \alpha_1 \mu + \omega \alpha_1 \mu - r_1 \alpha_2 \mu + \alpha_1 \beta \mu - \alpha_2 \beta \mu + \alpha_1 \mu^2 - \alpha_2 \mu^2}, \\ B &\to \frac{N_1 \alpha_1 \beta \lambda - r_1 \beta \mu - \beta^2 \mu - \beta \mu^2}{\omega \alpha_1 \beta + r_2 \alpha_1 \mu + \omega \alpha_1 \mu - r_1 \alpha_2 \mu + \alpha_1 \beta \mu - \alpha_2 \beta \mu + \alpha_1 \mu^2 - \alpha_2 \mu^2} \end{aligned} \tag{4c}$$

The Eq. 1 system is nonlinear and shows different dynamical behaviors according to the OA model's equilibrium points. Jacobian matrix can be implemented to determine the stability of equilibrium points. The eigenvalues of the Jacobian matrix determine the behavior around the fix points of the OA model. Especially, the eigenvalues give information about the dynamic of the OA model near the equilibrium points in the phase space. They also help us to understand the dynamic whether chaotic or periodic. A Jacobian matrix is an essential component for numerical algorithms to solve non-linear inverse problems. Thus, the Jacobian matrix is implemented to the system to ensure linearity of the Eq. 1 as follows:

Parameters	Values
λ	0.0045
μ	0.0052
α_1	0.01338
α_2	0.00612
N	13.64
N_1	12.55
β	0.0098
ω	0.0011
r_1	0.096
r_2	0.039

Table 2. The parameters values of occupational accident model.

$$J = \begin{bmatrix} -A\alpha_1 - B\alpha_2 - \mu & r_1 - X\alpha_1 & r_2 - X\alpha_2 \\ A\alpha_1 & -r_1 + X\alpha_1 - \beta - \mu & 0 \\ B\alpha_2 & \beta & -r_2 - \omega + X\alpha_2 - \mu \end{bmatrix} \tag{5}$$

The eigenvalues (λ_1, λ_2, λ_3) of the Eq. (1) system were calculated using the Jacobian matrix ($|J - \lambda I| = 0$) for each OA model equilibrium point. The eigenvalues (λ_1, λ_2, λ_3) corresponding to equilibrium points are given in Table 3.

According to Table 3, the eigenvalues for the first equilibrium point λ_1, λ_2, and λ_3 are real. Though, λ_1, λ_2 are positive, λ_3 is negative. In this context, the characterization of the first eigenvalue is the saddle point. The eigenvalues for the second equilibrium point λ_1, λ_2, and λ_3 are negative and real. Therefore, the characterization of the second eigenvalue is the node. The eigenvalues for the last equilibrium point, λ_1 is real, λ_2 and λ_3 are imaginary. λ_1 is positive, λ_2's real part is negative and the imaginary part is positive, and λ_3's parts are negative. Thus, the last eigenvalue characterization is the spiral saddle point. The stability of three equilibrium points are stable–unstable manifolds for the first one, stable nodes for the second one, and stable spiral saddle and unstable manifold for the third one.

Computation of reproduction number $\Re_0$

There are some methods to obtain $\Re_0$, such as the definition method, the next-generation matrix-based method, and the epidemic curve and serial interval (SI)-based method in the literature[40]. This paper applies the next generation matrix method to obtain $\Re_0$[41–43]. The OA model linear stability of the accident-free equilibrium (*OFE*) can be determined by using $s(F - V)$ Jacobian matrix, where *s* is the maximum real part of the matrix. *F* and *V* are a non-negative and non-singular matrix, respectively. $\Re_0$ is related to the *s*. *OFE* is found $(X_0, 0, 0, 0)$ for the OA model. X_0 is the initial value of *X*. The Jacobian matrix can be obtained as below if the *OFE*s are substituted to the Jacobian matrix in Eq. (5),

$$J = \begin{bmatrix} -\beta - \mu - r_1 + \frac{\lambda N_1 \alpha_1}{\mu} & 0 \\ \beta & -\mu - \omega - r_2 + \frac{\lambda N_1 \alpha_2}{\mu} \end{bmatrix} \tag{6}$$

where $\frac{\lambda N_1}{\mu} = X_0$. The accident compartments are *A* and *B*. *F* and *VOFE* matrices can be written concerning the Hurwitz Theorem as

$$F = \begin{bmatrix} \alpha_1 X_0 & 0 \\ 0 & \alpha_2 X_0 \end{bmatrix} \tag{7}$$

$$V = \begin{bmatrix} \beta + \mu + r_1 & 0 \\ -\beta & \mu + \omega + r_2 \end{bmatrix} \tag{8}$$

The next generation matrix can be found as,

$$FV^{-1} = \begin{bmatrix} \frac{1}{\beta+\mu+r_1} & 0 \\ \frac{\beta}{(\beta+\mu+r_1)(\mu+\omega+r_2)} & \frac{1}{(\mu+\omega+r_2)} \end{bmatrix} \tag{9}$$

The next generation matrix has two eigenvalues $\lambda_1 = \frac{N_1\lambda_1}{\mu(\beta+\mu+r_1)}$ and $\lambda_2 = \frac{N_1\lambda_2}{\mu(\mu+\omega+r_2)}$. λ_1 and λ_2 can be found 1.304914 and 1.46726, respectively, if the value of parameters in Table 2 are substituted to the eigenvalues. The largest absolute value of eigenvalues gives the R_0 reproduction number that is

$$R_0 = \frac{N_1\lambda_2}{\mu\,(\mu + \omega + r_2)} \tag{10}$$

Once $\Re_0 < 1$, the number of people who had an accident decreases to 0 monotonically; while if $\Re_0 > 1$, then this number will increase. In our case, the dominant eigenvalue has a positive value, which means the OA model is unstable.

Equilibrium points(X*,A*,B*)	Eigenvalues			Characterization	Stability
	λ_1	λ_2	λ_3		
(10.8606, 0, 0)	0.0343148	0.0211669	− 0.0052	Saddle point	Stable–unstable manifold
(7.40196, 0, 2.85473)	− 0.015628	− 0.0119618	− 0.00704292	Node	Stable
(8.29596, -2.19186, 3.92597)	0.017609 + 0.i	− 0.00601882 + 0.00438355i	− 0.00601882 − 0.00438355i	Spiral saddle point	Stable spiral saddle, unstable manifold

Table 3. Stability analysis of Occupational accident model of the equilibrium points[39].

Results

Numerical simulation and result of OA model

The OA model is a coupled first-order ordinary nonlinear differential equation. It is hard to find an exact solution due to nonlinearity. Therefore, the equation system (1) OA model is solved numerically by the Runge–Kutta method using parameters in Table 2 and used Matlab ODE45 routine[44,45].; Matlab ODE45 routine uses explicit Runge–Kutta(4,5) formula to solve ordinary differential equations given as below,

$$y'(x) = f(x, y), y(0) = y_0 \tag{11}$$

here $f : \mathfrak{R} \times \mathfrak{R}^m$ and $y_0 \in \Re^m$ represents initial vector value. The formula of the explicit Runge–Kutta method for *s*-stage with step size *h* is given as

$$y_{n+1} = y_n + h \sum_{i=1}^{s} b_i k_i \tag{12}$$

where

$$k_i = f\left(x_n + c_i h, y_n + h \sum_{j=1}^{i-1} a_{ij} k_j\right), i = 1, 2, \cdots s \tag{13}$$

c_i can be described as

$$c_i = \sum_{j=1}^{i-1} a_{ij} \tag{14}$$

and the Butcher tableau[46] for general implicit Runge–Kutta formula is given in Table 4.

The initial condition are chosen in January 2013 occupational accident data which is (X(0), A(0), B(0), Z(0))= (13.639 0.007713 0.005874 0.000108). All initial conditions are scaled 1/1000000. The equations system is solved in 96 steps with a time interval of a month. The numerical results are given in Fig. 3.

The normalized mean square error (NMSE) method is used for the OA model of operability. The NMSE method is which calculates the error by comparing the real data with the numerical solution. The method is given as follows[48];

$$NMSE = \frac{\Sigma(g(x, y)) - f(x, y))^2}{\Sigma(f(x, y))^2} \tag{15}$$

where $g(x, y)$ and $f(x, y)$ represent the real data and the numerical solution, respectively. The error of the OA model is given as Table 5.

Numerical simulation and result of modified OA model

In this section, a new parameter is added to the OA model, and its effects on the dynamics of occupational accidents are investigated. The effectiveness of occupational health and safety training in reducing the number of occupational accidents was taken into consideration during the selection of this parameter. In addition, taking OHS re-training for every person who had an occupational accident is mandatory by regulations in Turkey. A modified OA model is developed to determine the OHS re-training effect on OA model dynamics by adding *u* parameters. The addition of the parameter *u* is illustrated in Fig. 4.

where *u* represents the ratio of OHS re-training. While *u* has a positive sign due to positive contribution where added to r_1 and r_2 parameters according to legal regulations, *u* has negative for *Z* variable due to mortality. The Eq. (16) system of the modified OA model is given as:

$$\frac{dX}{dt} = \lambda N_1 + (r_1 + u)A + (r_2 + u)B - \alpha_1 XA - \alpha_2 XB - \mu X \tag{16a}$$

0					
c_2	a_{21}				
c_3	a_{31}	a_{32}			
⋮	⋮		⋱		
c_s	a_{s1}	a_{s2}		$a_{s,s-1}$	
	b_1	b_2		b_{s-1}	b_{s-1}

Table 4. General explicit Runge–Kutta Butcher tableau[47].

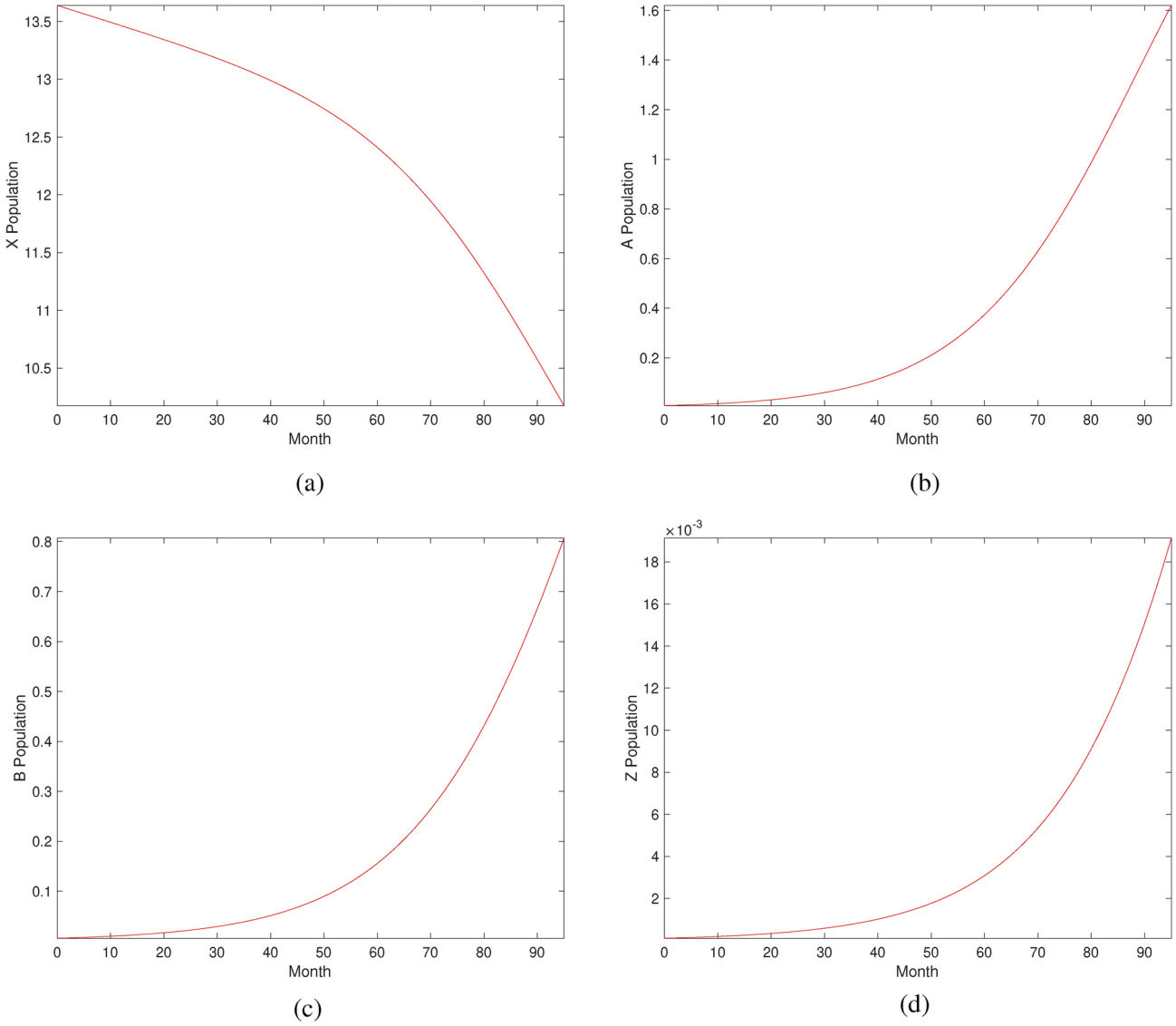


Fig. 3. Numerical simulation of occupational accident (OA) model, (**a**) The OA model numerical result of the number of insured without accident (Population X), (**b**) The OA model numerical result of the number of insured persons who had accidents reported within 3 days or less (Population A-Minor accident), (**c**) The OA model numerical result of the number of insured persons who had reported more than 3 days (Population B-Major accident), (**d**) The OA model numerical result of number of insured persons who had a fatal accident (Population Z-Fatal accident).

Variable	Values
X	3.9×10^{-6}
A	1.7×10^{-3}
B	2.8×10^{-3}
Z	2.6×10^{-3}

Table 5. Obtained NMSE values for X, A, B, Z variables.

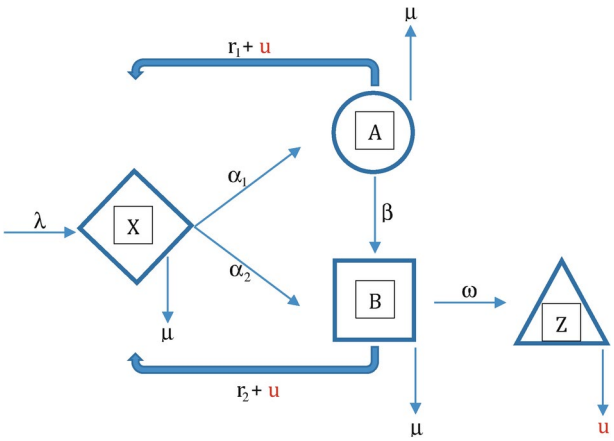


Fig. 4. Visual representation of the modified occupational accident model structure with parameter u added.

$$\frac{dA}{dt} = \alpha_1 XA - (\mu + (r_1 + u) + \beta)A \tag{16b}$$

$$\frac{dB}{dt} = \alpha_2 XB + \beta A - (\mu + (r_2 + u) + \omega)B \tag{16c}$$

$$\frac{dZ}{dt} = \omega B - uZ \tag{16d}$$

Although it is obligatory for all employees who have had occupational accidents to take OHS re-training before returning to work, such re-training may be delay or not processed at all due to certain difficulties such as lack of time, experts and financial resources. We assume that the re-training rate might be less than 50% in Turkey's OHS services. Thus, the u value is selected as 0.006 (60% of them take OHS re-training) to investigate decreases in the number of occupational accidents. The equation system 16 for the modified OA model is solved numerically by the Runge–Kutta method using Table 2 parameters with $u = 0.006$ and used Matlab ODE45 routine. The initial condition for each variable are chosen from January 2013 work accident data. The equation system is solved for 108 steps with a 1-month time interval. The numerical results for Eq. (1) and (16) systems are given in Fig. 5.

In Fig. 5, the decreasing effect of parameter u on populations A, B, and Z is demonstrated. The disparity between the two lines becomes more pronounced after the $40th$ month, and it further increases after the $96th$ month. The subtracted numerical results between the original OA model and the modified OA model (after $96th$ month) are provided in Table 6. The Numerical results indicate a decrease of 24% in population A, 33% in population B, and 40% in population Z, which is observed by the $108th$ month.

Conclusions

The primary objective of this research is to evaluate the number of occupational accidents and the preventive measures' impact on the dynamics of occupational safety, as viewed through the perspective of the first ordinary differential equation.Therefore, monthly SSI data between 2013 to 2021 are used to analyze the dynamics of the occupational accident model. The OA model's dynamics have been demonstrated with four variables and each variable's relations are described with eight parameters. While the parameters β, r_1, r_2, and ω have been found through fitting to the real SSI data, the others have been calculated by using SSI and announced TUIK data. The OA model has been solved numerically by using the Runge–Kutta method. Numerical results for the OA model shown in Fig. 3 are compatible with real data. The error between the OA model and the real data has been derived by using NMSE. The result for population X, A, B, and Z are 3.9×10^{-6}, 1.7×10^{-3}, 2.8×10^{-3} and 2.6×10^{-3}, respectively. Also, the OA model has been modified by adding u parameter regarding to OHS re-training. Within the modified OA model, minor, major, and fatal accident rates exhibit a decrease compared to the numerical results of the initial OA model, highlighting the importance of providing OHS re-training to individuals who have experienced occupational accidents. At the end of the $108th$ month, decrease in population A, B and Z are 24%, 33% and 40%, respectively. Despite requiring all employees who have experienced occupational accidents to undergo OHS re-training before resuming work, the process may be delayed or incomplete due to various challenges, including time constraints, lack of expertise, and financial limitations. In this study, based on the numerical results we have obtained, recommendations are provided to the government to increase the frequency of workplace inspections related to OHS training for employees and to establish regulations concerning penal sanctions. The OA model can also be applied to predict the number of occupational accidents and implement preventive measures using data from other countries. Moreover, it can be adapted for high-risk sectors such as industry, construction, and mining. New prevention parameters could be developed in these areas to enhance safety measures. In addition, our future plans include applying the recommended regression analysis and deep learning methodologies to analyze the dataset.

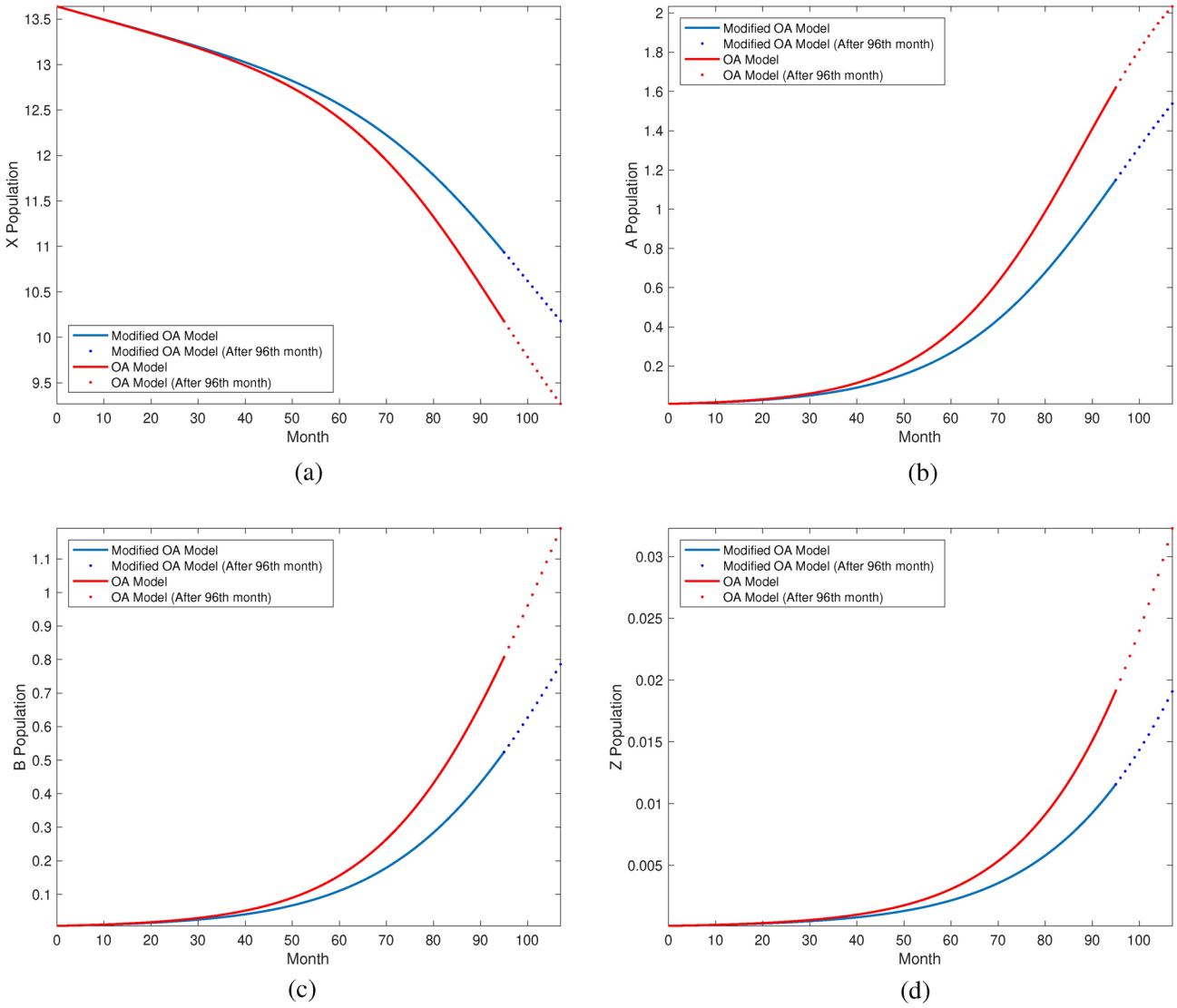


Fig. 5. Compare the numerical results of the OA model with the results of the modified OA model, (**a**) The OA model and modified OA model numerical result of the number of insured without accident (Population X), (**b**) The OA model and modified OA model numerical result of the number of insured persons who had accidents reported within 3 days or less (Population A-Minor accident), (**c**) The OA model and modified OA model numerical result of the number of insured persons who had reported more than 3 days (Population B-Major accident), (**d**) The OA model and modified OA model numerical result of number of insured persons who had a fatal accident (Population Z-Fatal accident).

Population A-(OA model)	Population A-(Modified OA model)	Population A-(subtracted result)	Population B-(OA model)	Population B-(Modified OA model)	Population B-(subtracted result)	Population Z-(OA model)	Population Z-(Modified OA model)	Population Z-(subtracted result)
1.6606	1.1833	0.4774	0.8371	0.5442	0.2929	0.0200	0.0121	0.0080
1.7002	1.2169	0.4833	0.8675	0.5643	0.3031	0.0210	0.0126	0.0084
1.7389	1.2505	0.4884	0.8983	0.5849	0.3134	0.0219	0.0132	0.0088
1.7766	1.2839	0.4927	0.9295	0.6058	0.3237	0.0230	0.0138	0.0092
1.8133	1.3171	0.4962	0.9611	0.6272	0.3340	0.0240	0.0143	0.0096
1.8488	1.3500	0.4988	0.9931	0.6489	0.3443	0.0251	0.0150	0.0101
1.8830	1.3826	0.5005	1.0255	0.6709	0.3545	0.0262	0.0156	0.0106
1.9160	1.4147	0.5013	1.0581	0.6934	0.3647	0.0273	0.0162	0.0111
1.9476	1.4464	0.5012	1.0910	0.7161	0.3749	0.0285	0.0169	0.0116
1.9778	1.4775	0.5003	1.1241	0.7392	0.3850	0.0297	0.0176	0.0121
2.0066	1.5080	0.4985	1.1575	0.7625	0.3950	0.0310	0.0183	0.0126
2.0338	1.5379	0.4959	1.1910	0.7861	0.4048	0.0323	0.0191	0.0132
2.0595	1.5671	0.4925	1.2246	0.8100	0.4146	0.0336	0.0198	0.0138

Table 6. Subtracted numerical results between the OA model and the modified OA model (after $96th$ Months) (Scaled = 1/1000000).

Data availability

The authors declare that the data supporting the findings of this study are available within the paper, its supplementary information files, and Turkish Social Security Institution. (https://www.sgk.gov.tr/Istatistik/Yillik/fcd5e59b-6af9-4d90-a451-ee7500eb1cb4/).

Code availability

The codes used in the numerical solution of the OA model, the modified OA model, and parameter fitting are available at the following link: https://bilgiedutr-my.sharepoint.com/:u:/g/personal/selcan_cicek_bilgi_edu_tr/EbYCBtKqdSNBjlABYj9I2g4BkxQODTpmc2lkQdjL9mgw4w.

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Author contributions

SK: Writing—original draft, Data Curation, Methodology, Software, Visualization. ET: Methodology, Software, Supervision, Conceptualization. ?E: Writing—review and editing, Supervision, Formal analysis.

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Competing interests

The authors declare no competing interests.

Consent to participate

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Consent for publication

The paper is submitted with the consent of all listed authors.

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