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Modeling and Optimizing Mileage Homogeneity in Public Transportation Using Gini Index

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ABSTRACT

Fleets of city buses experience uneven allocation of miles, which causes uneven wear, elevated maintenance expenditures, and inefficiencies when providing service. This research proposes the application of the Gini index as a base-objective function for attaining a higher level of homogeneity of miles traveled by fleets. Using 756 days of operating data from 160 articulated public buses serving Makkah, Saudi Arabia, the research initially assesses the scope of each day's mileage inequalities using descriptives and Lorenz curve interpretation. The results suggest unevenness during the early operations significantly, whereby the Gini attains a maximum of 0.38, diminishing as allocation tactics by heuristics were undertaken. To further reduce the level of unevenness, an integer model employing the Gurobi optimizer was formulated for Python. Used for a subset of 25 buses, the model reduced the Gini from a level of 0.1648 (mean) down to a level of 0.0462, exhibiting a significant improvement regarding fairness of work. Beyond the numeric contribution, the research results call attention further to operational merits, such as level maintenance scheduling, expanded lifetimes for vehicles, and increased sustainability. While the analysis focuses on a single large-scale urban bus network, the proposed framework can be extended to multi-route and multi-modal systems, incorporating demand variability and additional equity dimensions in future research. By integrating the Gini index as a transport planning objective into the vehicle assignment problem naturally, the research connects transport justice with the efficiency of operations, providing transport agencies with a functional model for sustainable and equitable bus operations.

Keywords: Gini Index, Optimization, Equity, Homogeneity, Public Transportation, Maintenance

Introduction

Public transportation systems are the backbone infrastructure components in the centers of contemporary cities, which provide environmentally sustainable mobility options. Due to growing urban population and urbanization, the efficiency and effectiveness of public systems have become more significant. By serving multiple types of users with multiple daily trips, public transportation offers various advantages including decreasing traffic congestion, decreasing air pollution, and maximizing the use of energy.

The efficient operations of public transport networks rely on the smooth interface of various parts. The bus fleet is one of the most fundamental parts of such networks. Comprising the heart of the urban transport network, the city's bus fleet supports the daily travel hundreds of thousands of people. Managing such a fleet is inherently complex and multi-dimensional. Passenger demand, routing, maintenance requirements, and driver management, among other variables, need to be carefully considered in order to keep the operations efficient and sustainable.

Thus, among the major issues in bus fleet operations is the inconsistency in the release of daily mileage. On any fleet of vehicles, some vehicles cover long distances in the day while others cover relatively small routes. The inconsistency results in disproportionate wear and tear of vehicles as well as rising maintenance costs. More frequent maintenance with vehicles because of high use doesn't only add the cost of operations but also extends the vehicles' downtime. Such inconsistency is harmful in disrupting the fleet's overall efficiency as well as the efficiency of public services.

Achievement of homogeneity in the miles traveled by bus fleets - that is, ensuring as much as possible that all the buses run the same number of miles - is one significant goal of efficient fleet management. Achievement of the goal is maximally advantageous in terms of vehicles wear and tear, maintenance costs, and uniformity in services due to minimized downtime. In addition to that, fair and effective utilization of available resources is also vital in the attainment of sustainable public transport networks.

The study recommends the use of the Gini Index as a way of facilitating more equitable mileage redistribution across bus fleets. Whilst the Gini Index may be most famously associated with the measuring of income inequality, it is no less applicable when exploring how well distributed are the assets. By measuring inequalities in a dataset, it gives an clear numerical

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insight into inequality—the contrast in kilometers travelled across the many different buses. The use of this index across public transport allows possible detection of imbalances whilst also informs the development of certain measures aimed at more equitable division of daily travel. Through this, it thus fosters more efficient working as well as more equitable use of assets.

The paper mentions the use of the Gini index as an aid in the achievement of consistency in mileage in bus fleets. Though in general used as an index of income inequality, the Gini Index is just as accurate in the measurement of the equitability in the distribution of the resources. Being used in the measurement of the differences in the data set, the Gini index provides us with the numeric value of the asymmetry of the kilometers traveled distribution of the vehicles. Through the use of the index in public networks, there is the ability in the design of efficient measures that would provide maximum daily distribution of the mileage, the end result improving the efficiency in the operations as well as the equitability in the available resources.

The purpose of this research is to offer an innovative way of improving the homogeneity of bus fleet mileage using the Gini Index. The analysis in this context will be focused on the examination of the current homogeneity of vehicle mileage distribution, the estimation of the Gini index, and the use of optimization models. The study will use this method with the objective of helping in the more effective and environmentally sustainable management of bus fleets.

Even as the literature on equity in transit continues to expand, significant areas for contribution exist in the operationalization of fairness in fleet administration, especially in the allocation of mileage among vehicles.

While equity was reviewed in a comprehensive manner in the case of accessibility, travel time, and cost, the issue of mileage equity in bus fleets was not considered much. Previous studies overwhelmingly use the Gini index descriptively, although use as an objective of optimization has been sparingly studied. Moreover, most such studies are small in scale or theoretical in nature, with limited exploration on practical transit networks. Filling in the blanks, the current study embeds the Gini index as a primary objective in a block–vehicle task assignment problem for articulated buses. Using data from Makkah for a duration of over two years, the problem establishes through the model that mileage equity improvement not only maximizes fairness but also benefits maintenance scheduling, fleets longevity, and sustainability in transit operation. While thus filling the gap between transport justice on the one hand and operation optimization on the other hand, the study provides practical insights for resource allocators who are seeking balanced resource distribution.

This study starts with a literature review on transportation equity and corresponding measurement approaches and introduces a Gini index-oriented system for measuring and optimizing homogeneity in bus mileage distribution. Data collection processes are made a part of methodology and extend the usability of Gini index through empirical data from Makkah including the computation of 756 Gini values for a duration of two years in an attempt to understand temporal distribution patterns. An integer program is then defined, comprising well-established objective function, constraints, and decision variables. Formulation is then applied on a selected bus route of 25 vehicles, programmed in Python via Gurobi solver, and optimized via Branch-and-Bound algorithm. Annual optimal assignment plan identifies a significant minimization of Gini index, favoring equity in distribution of fleets.

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Research Problem and Objectives

Achieving balance in the assignment of buses to service operations in public transportation systems is essential. When such balance is not maintained, maintenance planning becomes more challenging and may result in additional costs for operating companies. In an environment where cost reduction is a priority and competition is increasing, the efficient and balanced utilization of resources is of critical importance. Even though equity and sustainability are becoming more and more important in public transportation planning, operational equity in bus fleet management—specifically, the fair distribution of vehicle mileage—is still not given enough attention. Therefore, the main research issue that this study attempts to solve is: How can equity in bus mileage allocation be explicitly operationalized and optimized through a rigorous mathematical framework in public transportation systems?

This study aims to achieve three goals:

(1) To use the Gini index to measure mileage inequality,

(2) To incorporate the Gini index in mathematical and optimization models,

(3) To assess the effects on sustainability and operations using actual data.

Literature Gap

An examination of the existing literature on the Gini index indicates that prior studies have predominantly focused on the expansion of public transportation services or on the evaluation of service levels. However, there is a notable lack of research examining resource efficiency and balance in public transportation planning. The majority of studies treat the Gini index as a descriptive indicator rather than directly incorporating it into optimization models, despite the substantial expansion of the literature on transport equity. Furthermore, in large-scale, actual public transportation systems, equity in vehicle-level operational workload—particularly mileage distribution among buses—remains largely unexplored.

To the best of the authors' knowledge, no previous study has used the Gini index as the main objective function for optimizing the mileage of a large bus fleet under actual operational constraints.

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Contributions of This Study

There are four main ways in which this study adds to the literature:

- It presents the Gini index as the main objective function for homogeneity in bus mileage.
- It incorporates equity directly into a Gurobi-solved branch-and-bound integer programming model.
- Using data from 160 articulated buses over 756 days, it offers extensive empirical evidence.
- It shows how equity-oriented optimization enhances asset longevity, sustainability, and maintenance planning.

Although this study is conducted within the public transportation sector, the proposed approach can also be adapted to other sectors, such as private transportation and logistics.

Literature Review

Three primary themes serve as the framework for the literature review:

- transportation equity and sustainability in urban planning;
- quantitative equity measures in transportation systems; and
- approaches to equitable resource allocation that are focused on optimization and policy.

Urban transport networks were identified as integral for sustainable city development due to their contribution towards easing congestion, lessening environmental impact, and enhancing mobility. In recent years, more prominence has been accorded towards including equity into transport planning paradigms as part of the transition from efficiency-based planning for transport systems towards more balanced transport service allocation between population subgroups[1,2]. Poorly designed transport networks have been shown as perpetuating spatial as well as socioeconomic inequities, whereas holistic mobility policies covering environmental, socioeconomic, as well as technological factors garner promotion[1,2]. Needs for planning with a focus for equity also manifested for studies covering optimization as well as spatial planning. Multi-objective models covering combination of fairness as well as operational goals were formulated[3] , whereas GIS as well as genetic algorithms used for solving spatial injustices as well as boosting diversity-accessibility[4]. Mathematical models such as proportional as well as alpha fairness provide formal methods for distributive fair resource allocation[5]. Theories of ethics including those by Rawls as well as utilitarians also used for planning frequencies for the purposes towards boosting fairness towards provision for transport provision[6]. Fears of unwarranted exclusion from the benefits of digital innovation due to the potential exclusion for the digitally disconnected or disadvantaged by new innovations for mobility[7,8] raised. Amendments from the concept of the concept of justice for the purposes towards compensating for the shortcomings of the traditional utilitarians for the vulnerable[9–11] put forward. Similarly, the potential negative impact of investments for transport structure for the purposes worsening the unequal yardstick used[12–14] highlighted for the purposes towards advocating for policy protection measures. Transport planning for transport planning also been correlate with the political, the institution-based, as well as the geographic factors as a reminder whereby provision of access by itself does not translate into fairness[15–18]. The mode of public transport provision for transport provision been shown as long-run socio-spatial implication-yielding but balanced delivery of gains as well as charges not easily effected as long as the spatial as well as population-based disparities persists[19,20] . The built environment has been shown to importantly affect mobility equity. The nonlinear and heterogeneous impacts of the distribution of jobs, dwelling, and servicing on travel demand have been revealed[21], as data-driven researches been proven indispensable for evaluating imbalances between older and disadvantaged groups[22]. Further definitions of transport justice further extend into concerns such as crowding exposure and safeties by gender, highlighting user experience, reliability, and quality over mere provision of service[23,24]. Collectively, these latter findings lend support for the use of the equity-aware goals for transport system optimization.

These studies offer a robust conceptual framework for equity and sustainability in urban transport planning; however, they also underscore the necessity for quantitative indicators capable of systematically measuring, comparing, and monitoring inequality within operational public transportation systems.

Recent studies on urban transportation planning indicate that considerations of equity must be addressed not only in the formulation of long-term strategies and policies but also in daily decision-making processes[25]. Real-world research reveals that when planning goals and daily operations don't match up, service delivery can be less efficient and less fair[25,26]. For instance, research on large-scale urban transportation has demonstrated that congestion patterns, service interruptions, and uneven infrastructure utilization result not solely from demand, but also from operational allocation strategies and policy limitations[25,27] (e.g., network assessments emphasizing congestion and performance evaluations attuned to policy). These

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findings bolster the notion that planning principles incorporating fairness ought to be integrated into operational optimization frameworks, rather than merely serving as measures for assessing past occurrences[28].

Recent studies have increasingly demonstrated the significance of operational-level decisions in maintaining long-term transport equity and system resilience from both policy and sustainability perspectives[26,29]. Studies on transport planning policies and sustainability transitions indicate that the geographical and temporal allocation of operational resources significantly influences the equity of service distribution, resource use, and accessibility outcomes[25,26]. Adding policy-based equity goals to quantitative optimization models is increasingly considered as an important step toward making sure that strategic transport planning goals match up with how things work in the real world[28]. This growing corpus of research stresses the need for optimization approaches that clearly take fairness concerns, like fair service supply and task distribution, into account when figuring out how to manage public transit[25,28].

Mileage allocation has been revealed as a central fairness metric for public transport. Current inequalities for large cities have been revealed by the Gini index[30], as GIS-based optimization used for increasing the frequencies of disadvantaged area usage of the bus[31]. The Gini and Theil indices revealed how travel time and cost burden disproportionately fall upon minority groups[32]. Need-based allocation shown by alpha-fairness methods correspond with the spirit of the mobility justice[5]. The Gini and Theil indices also revealed how spatial equity quantified by Gini coefficients reveals the varying fairness profiles by mode[33]. Combinations between weak usage of the transit and the locales of high imparity further suggest the inefficiency of the mismatch between the resources[34]. Demands-supply imbalances as well as accessibility gaps for carless citizens also exist[35,36]. Demographic factors such as the ages, the genders, and the incomes importantly affect the accessibility outcomes[37,38]. The routing and the network circuity also further extend the time inequalities[39,40]. Wider considerations of the equity reach the policy fairness of the environment[41], the subsidy inefficiencies[42], as well as multi-modality equality for user choice[43]. Additional research examined access bias from loyalty programs[44],COVID-19 service cuts creating inequities[45], and the demand-responsive service potential for reducing sparsely populated region inequities[46]. The uneven allocation of vehicle kilometers traveled (VKT) became evident as a cause of system imbalances, as over-interested regions prove inefficiencies and underserved places affect vulnerable populations such as older adults and women disproportionately[22,24]. Service frequency fluctuations at the neighborhood level were revealed as providing unequal benefits[47]. Homogeneity of VKT thus comes as a way of increasing simultaneously the equity and the efficiency. Additional indicators, namely the Suits index and spatial regression tools, were used for estimating crowding in vehicle, thus further linking inequities with broader socioeconomic segregation[23]. Spatiotemporal variability of travel behavior became emphasized as a needed component of the study of equities[48]. The Gini index, originally describing income inequality, came revealed amenable for the study of accessibility disparities, travel time allocation, and allocation of the public transport service[49]. Indices applications vary from travel time and cost inequities[32,50] up to spatial research into equities[33] and research into arising transport networks such as self-driving cars[51]. Scalability became demonstrated for multi-city studies through the means of the GTFS[30]. Recent research included the Gini index into multi-objective schemes, thus enabling the estimation of impacts of delay as well as optimality of resource allocation[52,53]. The index also been used for the study of subsidies allocation[36], scheduling of the flow[54], as well as the equilibrium between fairness and the equilibrium between efficiency[11]. Index results were demonstrated as affected by the level of the power, the city scale, as well as the level of the development[55,56]. At the regional level, the transport investment fairness between the counties were compared[57]. Inequities for disadvantaged populations also have been found, including access gaps for the disabled[58], age and gender gaps[59,60], and temporary reductions of inequality for the investments in high-speed rail[61]. Risks of marginalization due to incompletely integrated multi-modal systems also came into focus[62].

Recent studies depict the evolution of the Gini index from a descriptive into a prescriptive indicator, which more regularly impacts choices for investment into infrastructures, modification of services, and fare policies. Aspirations for time and monetary cost were shown to highly alter Gini outcomes, thereby unearthing hidden inequities[19].

Nonetheless, most equity metrics, like the Gini index, are mostly used to describe or evaluate things, not to make decisions that are focused on optimization. This is especially true in large, policy-driven public transportation systems.

Although numerous equity indicators have been suggested in transportation literature, including entropy-based measures like the Theil index, need-based approaches such as alpha-fairness, and variance-oriented distribution metrics, their direct incorporation into extensive operational optimization models is still constrained. Entropy-based indices frequently necessitate logarithmic adjustments and display susceptibility to outliers, complicating their integration into mixed-integer formulas. Likewise, alpha-fairness methodologies rely on inequality aversion parameters, necessitating further calibration and subjective assumptions. Variance-based measures, while computationally less complex, predominantly measure absolute dispersion and may not adequately represent relative disparity in diverse fleet contexts. Conversely, the Gini index offers an advantageous equilibrium among interpretability, responsiveness to distributional inequality, and mathematical manageability. The formulation, grounded in pairwise absolute deviations, facilitates explicit linearization via auxiliary variables, permitting its direct use as an objective function in a mixed-integer linear programming context. The Gini index is particularly appropriate for equity-focused optimization in public transportation operations, where transparency, scalability, and repeatability are critical.

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In addition to its computational feasibility, the Gini index offers a theoretically unique viewpoint in contrast to dispersion metrics based on variance or entropy. Variance quantifies absolute dispersion from the mean, but the Gini index assesses relative inequality throughout the distribution, aligning it more closely with equity-focused interpretations. In fleet operations, the Gini index indicates proportionate workload disparities rather than solely numerical discrepancies. Thus, reducing the Gini index directly addresses structural inequality in workload distribution, providing a more explicit managerial understanding of equity and balanced resource usage.

In this study, Gini index is utilized for measuring homogeneity in bus kilometer distribution using a new equity-oriented objective function. Contrary to the common optimization methods where minimization of cost or maximization of efficiency is focused, the present paper seeks to strike a balance in operational load in harmony with spatial discrepancy principles.

The reviewed literature indicates that attaining equity in public transportation requires a comprehensive strategy encompassing policy understanding, quantitative evaluation, and operational decision-making. The insights directly influence the conceptual framework of this study, as equity is not solely an evaluative outcome but is actively incorporated into the optimization framework. The emphasis on equitable resource allocation and fair operational practices indicates that uniform vehicle mileage serves as the primary objective of fairness. This directly shapes the configuration of decision variables, constraints, and performance metrics within the proposed optimization model.

To the best of the authors' knowledge, no previous study that specifically targets mileage homogeneity under realistic operational constraints has operationalized the Gini index as a primary objective function within a large-scale, real-world bus fleet optimization framework.

Methodology

Methodological Framework

This study employs a systematic and transparent methodological approach to implement fairness in bus fleet management via mileage uniformity. The framework has four consecutive stages: (i) measurement of baseline inequality, (ii) formulation of the optimization model, (iii) derivation of the exact solution using a branch-and-bound algorithm, and (iv) comparative assessment of optimized and observed results.

Initially, actual operational data are utilized to calculate daily and cumulative Gini index values, establishing a baseline evaluation of mileage disparity throughout the fleet. This step determines the magnitude and temporal progression of imbalance before any optimization measures are implemented.

In the second phase, an optimization model is developed that incorporates mileage homogeneity, quantified by the Gini index, directly into the objective function. The proposed formulation diverges from traditional fleet optimization models that focus on cost reduction or efficiency enhancement by explicitly minimizing disparities in vehicle workload while adhering to operational feasibility constraints, such as comprehensive service block coverage, single-task assignment per vehicle per day, and annual mileage planning for each vehicle. To facilitate a precise solution, the intrinsically nonlinear Gini index formulation is converted into a corresponding set of linear constraints, yielding a mixed-integer linear programming (MILP) model.

During the third stage, the MILP model is resolved via a precise branch-and-bound technique executed via the Gurobi Optimizer. Ultimately, optimal outcomes are methodically contrasted with actual mileage distributions to assess the efficacy of the equity-focused methodology.

Data Collection

The data utilized for the current work is the mileage data corresponding to buses deployed for city public transport in the city of Makkah, Saudi Arabia. Since the city of Makkah hosts millions of visitors and pilgrims every year, the sustainability and efficiency of the city transport system are of paramount importance. In such a context, the effective management of buses' fleets is an important factor for managing the needs of visitors and residents.

The data were created with Intelligent Transportation Systems (ITS) software employed by the public bus company of Makkah, such as Automatic Vehicle Location (AVL) and Oracle platforms. The data gained from the systems were subsequently imported into Tableau, a business intelligence and reporting system. Tableau produces detailed reports of the daily mileage of the buses and therefore provides a good point of departure for general data analysis and vehicle fleet optimization studies. The data compiled include the cumulative mileage data for the complete operational life of the 160 buses. The data were recorded daily for each bus and were vehicle-wise. The main aspects of the data are as follows:

- **Date:** The date the data were collected.
- **Cumulative Odometer:** The total cumulative distance traveled by the bus for the day, presented in meters.
- **Vehicle ID:** Unique number assigned for all buses.

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- **Daily Distance Traveled (Kilometers):** Kilometers traveled by the bus on a daily basis.

The dataset will be employed for a variety of analytical and modelling purposes consistent with the project objective. Its complete coverage and high-granularity facilitate detailed analysis of the day-to-day bus fleet operations and mileage split. Furthermore, the 25-month duration of the data offers an adequate chronological framework for the integration of seasonal and periodic phenomena and hence for the examination of long-term trends of operations. Data collection process entails the following steps:

- **Importing and Data Extraction:** The daily bus mileage data were extracted from Tableau software and converted into an analytical format. To ensure sufficient data quality for analysis, 756 days of data covering the period from June 6, 2022, to June 30, 2024, were extracted.

  Table 1 presents a sample subset of the complete dataset, showing each bus's daily traveled distance and its total accumulated mileage up to that date. The leftmost column, labeled "Date," indicates the corresponding date. The codes listed at the top of the table represent individual buses. The field "CAN (KM)" refers to the operational distance covered by the respective bus on that day, measured in kilometers. The field "End Cumulative Odometer" represents the total cumulative distance traveled by the bus by the end of that specific date and is expressed in meters.

  A blank cell or a zero under "CAN (KM)" indicates that the corresponding bus did not operate on that day. For example, the articulated bus A003 traveled an operational distance of 169.63 kilometers on July 19, 2022. At the end of that day, its cumulative odometer reading was recorded as 11,202.53 meters. Conversely, the articulated bus A001 did not operate on the same day, and therefore, its corresponding cell was left blank.

Date	**A001**		**A002**		**A003**		**A004**	
	CAN*	ECO**	CAN	ECO	CAN	ECO	CAN	ECO
2022-07-19					169.63	11,202.53		
2022-07-20	0.72	1,940.93	0.33	1,402.98	129.39	11,331.92		
2022-07-21	344.09	2,285.02	167.23	1,570.21	219.12	11,551.04		1,636.87
2022-07-22	241.87	2,526.88	1.07	1,571.28	1.35	11,552.38		
2022-07-23	147.71	2,674.59	1.40	1,572.68	166.33	11,718.71		
2022-07-24	19.41	2,694.00		1,572.68	316.27	12,034.97		
2022-07-25		2,694.00		1,572.68	244.13	12,279.10		
2022-07-26			281.72	1,854.40	348.23	12,627.33		
2022-07-27		2,694.00	74.81	1,929.21	240.36	12,867.68		
2022-07-28	0.93	2,694.92	188.84	2,118.04	339.76	13,207.44		
2022-07-29			357.07	2,475.11	343.54	13,550.98		
2022-07-30	1.11	2,696.03	191.50	2,666.61	348.14	13,899.12		
2022-07-31	1.03	2,697.06	19.32	2,685.92	92.47	13,991.59		
2022-08-01			283.20	2,969.12	188.33	14,179.92		1,636.87
2022-08-02			357.86	3,326.98	166.29	14,346.21		
2022-08-03			298.04	3,625.02	95.96	14,442.17	144.37	1,781.23

* CAN(KM)
** End Cumulative Odometer

Table 1. Sample View of the Extracted Raw Dataset

- **Organization of Data:** There is an analysis of 160 articulated buses spanning 756 days whose evaluation is based on realized Gini values.

- **Data Cleaning:** The data were cleaned for missing, inconsistent, or incorrect records and were identified and corrected, which is an essential step for the reliability of the analyses. In the process, there were some identified outliers. Whether a recorded daily mileage value was an outlier or not was established by considering the operator company's highest recorded commercial speed on any one route. In the listed date range, the highest commercial speed was recorded along Route A4 at 33.55 km/h. Even if the bus were running non-stop for 24 hours along the same route, the highest daily distance achievable would be 805.20 kilometers. Taking this value as the threshold of reference, any of the mileage data beyond this threshold were reviewed, and one of the outliers was identified. That extreme value is 12,044.616 kilometers, and when compared with the benchmark of the highest speed of 33.55 km/h, it was established that these values were

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unrealistic and were therefore probably the outcome of software-related recording faults. In order to make the analysis precise and uniform, the above outlier was adjusted through the corresponding bus End Cumulative Odometer readings. In the buses where an outlier existed, the incorrect mileage was replaced through the computation of the difference between the lowest End Cumulative Odometer reading taken after the date of the outlier and the highest reading taken prior to the date of the outlier. Through such means, it was possible to replace the data for the outliers with the correct figures and thus only valid data were considered for the analysis.

This process of data acquisition offers the foundation for the research and facilitates the necessary analysis and modeling exercises that can homogenize the daily mileage distribution for the bus fleets. The data thus acquired will facilitate the determination of the GINI index and the development of strategies for sustainable and improved fleet management.

GINI Index

The GINI index is a mathematical index developed first for the purpose of assessing income distribution differences. It is a number that expresses inequality within a population as a quantity ranging from 0 (perfect equality) up to 1 (perfect inequality). In the context of public transport and fleet management, the GINI index is an excellent instrument for assessing inequality within the allocation of everyday mileage for buses. In such an example, the index is computed and provides interesting data for the purpose of attaining a more balanced utilization of buses and a more just allocation of resources.

In bus fleets for public transport, homogenization of mileage allocation is foremost, particularly for the purpose of the optimization of operational and maintenance spending, and for the enhancement of the quality of the service provided. Equalization of mileage utilization across buses forestalls over-utilization on individual buses and rebalances the repair and maintenance requirements across the fleet. The GINI index is an indicator of the extent of equalization of such allocation. The lower the GINI index, the more homogenously allocated the mileage is. A higher GINI index, however, is indicative of a very high degree of heterogeneity or inequality.

The GINI index is calculated by the following process:

- **Data Preparation:** Mileage records for each day will separately be collected and maintained for all buses. So that the data can be compatible for the GINI index measure, an Excel VBA program was developed.
- **Calculation of Cumulative Distribution:** Mileage of each bus is employed for the determination of its proportionate share of the total mileage, and hence a cumulative distribution chart is developed.
- **Lorenz Curve and Line of Equality:** The actual mileage distribution is illustrated by the Lorenz curve, while the line of equality represents the ideal condition in which mileage is evenly distributed among all buses. Figure 1 presents an example depiction of the line of equality and the Lorenz curve.

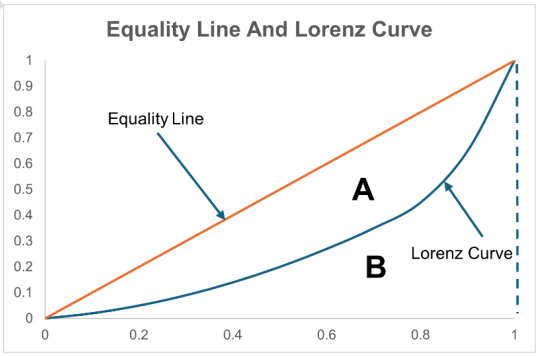


Figure 1. Equality Line and Sample Lorenz Curve

- **Calculation of the GINI Coefficient:** The GINI index is the result of the area under the curve of the Lorenz curve and the line of equality. It can be determined as the "gap" below the two curves and the GINI coefficient as the percentage of the gap with respect to the total below the line of equality curve. In other words:

$$Gini = \frac{A}{A+B} \tag{1}$$

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The homogeneity of mileage distribution within the articulated bus fleet was determined using the GINI index, which reached a value of 0.38144 on the 102nd day—identified as the day with the lowest homogeneity. Table 2 presents the calculation of the Gini value by showing both the area under the Lorenz curve and the difference between the line of equality and the Lorenz curve. This high value confirms that mileage usage across the fleet had been relatively heterogeneous up to that point.

An examination of the Lorenz curve further supports this observation. As illustrated in Figure 2, the Lorenz curve shows a clear deviation from the line of equality, indicating that a considerable number of vehicles bore a disproportionately high operational load. This situation suggests that no intervention had been implemented to maintain homogeneity in fleet utilization up to the 102nd day. At the same time, it can be inferred that there remains significant potential for improvement in promoting greater equity in mileage distribution.

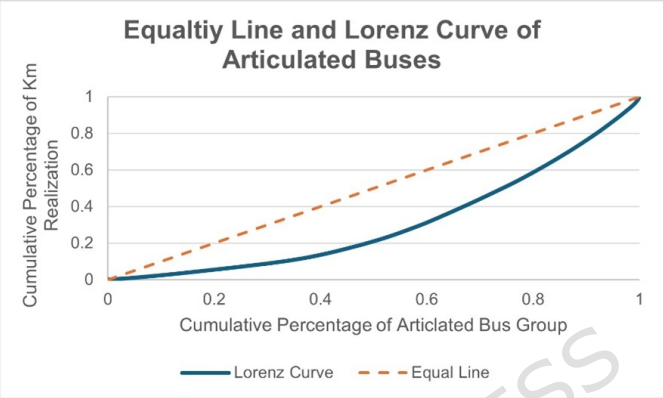


Figure 2. Equality Line and Lorenz Curve for Articulated Buses

Total Area Under Equal Line	Total Area Under Lorenz Curve (B)	Area Between Curve and Line (A)	GINI (A/(A+B))
0.5	0.309279656	0.190720344	0.381440687

Table 2. GINI Index Calculation for Articulated Buses

Data Analysis

This section presents the daily analysis of the articulated bus fleet and the evaluation of mileage allocation equity. The analysis combines descriptive statistics and inequality measures to examine the extent to which daily operations are evenly distributed among the buses.

Table 3 provides the descriptive statistical values of the dataset, which consists of 756 daily Gini index values. The daily analysis of the Gini index yields an average value of 0.1519, indicating that the overall mileage distribution among the buses is highly stable. The absolute minimum and maximum values were calculated as 0.0763 and 0.3814, respectively. This shows that, although the system experienced relatively higher inequality during certain periods, it generally maintained a reasonable level of balance.

Avg.	Std. Error	Median	Mode	Std. Dev.	Sample Var.	Kurtosis	Skew.	Range	Min	Max	Count
0.1519	0.0031	0.111	0.111	0.0853	0.0073	0.4202	1.2578	0.3051	0.0763	0.3814	756

Table 3. GINI Index Calculation for Articulated Buses

The descriptive statistics support this observation. The positive skewness (1.2577) indicates that lower Gini values are dominant in the distribution, though a few higher values pull the overall distribution upward. The low kurtosis value (0.42) suggests a relatively flat and homogeneous distribution of the Gini values around the mean.

The visualization of daily Gini index values in Figure 3 further illustrates these dynamics in detail. During the first 100 days, the Gini index shows a sharp increase, peaking at around 0.38. This reflects periods when some buses were used significantly more intensively than others, leading to temporary imbalances in mileage distribution. However, after this peak, the values gradually decline, reaching approximately 0.1 around the 400th day, and remain consistently low thereafter.

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This trend indicates that the operating company initially did not employ a structured vehicle assignment strategy, which caused the Gini index to rise to its maximum level of 0.38. This demonstrates how rapidly mileage inequality can accumulate in the absence of systematic intervention. Around the 100th day, the company introduced heuristic balancing strategies aimed at enhancing homogeneity, which led to a significant and sustained reduction in the Gini index.

A closer examination of the daily Gini trajectory reveals a clear structural shift in the system's operational behavior. After the introduction of balancing practices, the index follows a downward trend and eventually stabilizes at lower levels, suggesting the emergence of an operational equilibrium in mileage allocation. The long observation period therefore captures not only short-term fluctuations but also the dynamic adjustment capacity of fleet management policies over time. These temporal patterns indicate that mileage inequality is highly sensitive to allocation strategies and can be systematically controlled through structured interventions.

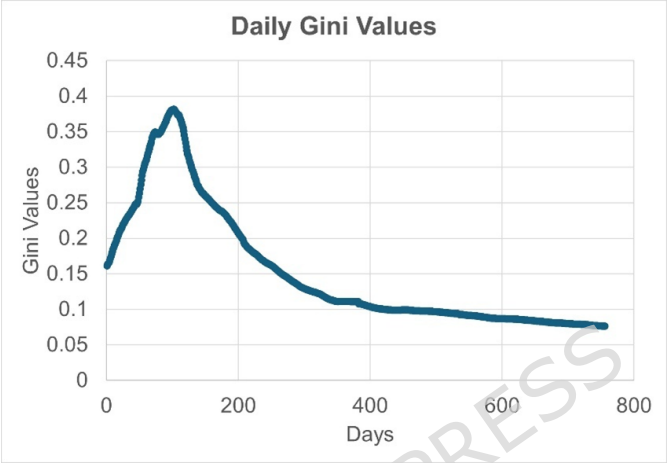


Figure 3. Daily Gini Index Values Graph

In conclusion, while the system experienced some variability within the first phase, the subsequent phases register an exceedingly high degree of equilibrium and fairness of mileage allocation. This implies that vehicle usage became increasingly evenly balanced with the passage of time, resulting in sustainable fleet operation through the enabling of balanced wear and tear, improved maintenance scheduling, and increased vehicle longevity.

GINI Index Based MILP

Public transit system management can be made more efficient by minimizing mileage usage differences among bus fleets. The GINI index can be an effective tool for evaluating mileage distribution differences among buses, and the index minimization is the key for more even mileage allocation. In this section, we deal with the possibility of incorporating the GINI index as a mathematical constraint and minimizing it afterward.

Though initially created as a statistical tool measuring the extent of income inequality, the GINI index can further be used for determining the homogeneity of the distribution of resources. The index varies from 0 up to the value of 1, and where 0 signifies complete equivalence and 1 signifies total inequality. In public transport systems, the GINI index can be used for evaluating and minimizing differences within the usage of mileage among buses.

Reducing the GINI index for public transport operations tries to make the mileage distribution for the entire fleet uniform, thus causing uniform vehicle wear and tear and evening out maintenance expenditure. The mathematical formulation for reducing the GINI index can be stated as below:

Parameters:

km_v	This parameter represents the distance associated with vehicle task v
e_i	This parameter denotes the existing accumulated mileage of bus i

Decision Variables:

$y_{bv} \in \{0,1\}$	if bus b is assigned to vehicle task v and 0 otherwise
$y_{bv} = 1$	if bus b is assigned to vehicle task v and 0 otherwise
$x_b \geq 0$	The total annual kilometers assigned to bus b
$x_{ij} \geq 0$	An auxiliary variable representing the absolute difference in total kilometers between bus i and j

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Objective Function:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} z_{ij} \tag{2}$$

The objective function minimizes the sum of the absolute differences of yearly total kilometers traveled for all vehicle pairs. It therefore minimizes the GINI coefficient and prefers evenly balanced workload (kilometers) allocation for the fleet.

Constraints:

Each block must be covered exactly once:

$$\sum_{b} \sum_{v} y_{bv} \cdot \text{Cover}[block, vehicle\ task] = 1 \quad \forall block \tag{3}$$

Each block must be covered by exactly one bus-vehicle task assignment. This constraint is enforced through a binary cover matrix, which indicates what blocks can be grouped into what vehicle tasks.

Vehicle tasks consist of blocks that satisfy two fundamental criteria:

- **Temporal Compatibility:** Non-overlapping blocks assigned within the same vehicle task are prohibited, such that the bus can efficiently transit from one task to the other.
- **Maximum Daily Distance:** The total distance covered by the blocks for a vehicle task assigned shall not be greater than the bus's maximum daily allowable mileage.

All the vehicle tasks that are feasible are created by all the feasible block combinations that satisfy the above two constraints. The resulting binary cover matrix then provides the mapping between blocks and vehicle tasks — with the value of 1 indicating that a particular block is included within a particular vehicle task.

In the matrix and related assignment constraint of the model, it ensures that a block is visited only once, by and only by one vehicle task.

Each bus can be assigned to at most one vehicle task:

$$\sum_{v} y_{bv} \leq 1 \quad \forall b \tag{4}$$

Each bus can be assigned only one vehicle task per day for daily operations, and such an assignment will be repeated over the planning horizon (365 days).

Annual distance accumulation:

$$x_b = \sum_{v} km_v \cdot y_{bv} \cdot 365 \quad \forall b \tag{5}$$

The annual kilometres for each bus are determined as the number of daily kilometres for the vehicle task allocated multiplied by the number of days for one year.

GINI linearization constraints:

$$z_{ij} \geq (x_i + e_i) - (x_j + e_j) \quad \forall i, j \tag{6}$$

$$z_{ij} \geq (x_j + e_j) - (x_i + e_i) \quad \forall i, j \tag{7}$$

These constraints linearize the absolute value calculations needed for the total kilometers' pairwise differences evaluation, both for the assigned kilometers and for the existing mileage ei and ej of buses i and j, respectively.

The overall objective is to program vehicle work for buses such that all tasks are executed exactly once, operational constraints are satisfied, and the resulting annual distance allocation among the fleet decreases the GINI index and therefore moves toward workload fairness.

By putting the Gini index into the optimization model's objective function, the proposed formulation makes it clear how equity issues are related to operational decision-making. Using binary bus-assignment decision variables makes sure that each

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operational unit is assigned in a way that is possible and doesn't overlap with any others. The rules in place make sure that the business can run in a realistic way, like by limiting how far each vehicle can go each day and how many vehicles are available.

To figure out the Gini index, you need to add absolute deviations. This is done by adding auxiliary continuous variables, which creates a mixed-integer linear programming structure. This formulation lets you directly minimize mileage inequality during the optimization process, instead of having to wait until later to do so. The model shows a clear and direct link between decision variables, equity measurement, and how easy it is to run the business. To add the absolute deviations needed to figure out the Gini index, auxiliary continuous variables are added. This makes a mixed-integer linear programming structure. This formulation allows for the direct minimization of mileage inequality during the optimization process, rather than having to wait for the evaluation to happen. Because of this, the model makes a direct and easy-to-understand connection between decision variables, equity measurement, and operational feasibility.

Algorithmic Framework and Solution Procedure

Integration of Decision Variables and Gini Index Linearization

The optimization model includes binary option variables that assign each bus to a feasible vehicle task, as well as continuous variables representing annual mileage accumulation. Incorporating the Gini index into the optimization framework necessitates the calculation of pairwise absolute differences between total annual bus mileages. Auxiliary continuous variables are utilized to linearize the nonlinearity of absolute value formulas.

This linearization transforms the original nonlinear Gini formulation into a reasonable mixed-integer linear framework, enabling the direct minimization of inequality during the optimization process rather than as a subsequent evaluation. Thus, concepts of equality are intrinsically linked to operational decision variables, ensuring that justice is maintained during optimization rather than subsequently.

Branch-and-Bound Solution Approach

The optimization problem is addressed using a branch-and-bound approach because of the binary assignment decisions involved. Branch-and-bound methodically investigates the solution space by recursively dividing it into smaller subproblems. At every node of the search tree, a linear programming relaxation of the MILP is resolved to provide a lower bound on the objective function.

When the solution at a certain node contravenes integrality criteria, branching is executed on one or more binary decision variables. Bounding criteria are then implemented to discard subproblems with lower bounds that surpass the best-known feasible solution. This procedure persists until the global optimum is discerned within a specified optimality tolerance.

Within the framework of this study, branch-and-bound is crucial as the integration of binary bus-to-task assignments and auxiliary variables implemented for Gini linearization cannot be addressed only by continuous optimization methods. The algorithm ensures global optimality while maintaining operational feasibility and equity goals.

Implementation via the Gurobi Optimizer

The branch-and-bound method is executed via the Gurobi Optimizer, which is tailored for addressing extensive mixed-integer linear programming challenges. Gurobi's presolve routines diminish model size by removing superfluous constraints and refining variable bounds, while cutting-plane techniques enhance the linear relaxation and expedite convergence.

Parallel node exploration significantly improves computational efficiency, allowing the solution to manage the combinatorial complexity posed by equity-based constraints. Solver performance metrics, such as solution duration and the number of nodes examined, are tracked to guarantee transparency and reproducibility of outcomes.

The proposed methodology integrates the branch-and-bound algorithm with the Gurobi optimization environment, establishing a clear and replicable connection among decision variables, equity measurement, and operational constraints, thus addressing issues of methodological transparency and reproducibility.

The computational difficulty escalates with fleet size because to the quadratic increase of pairwise Gini linearization variables; yet, solver efficiency can be optimized by decomposition methodologies, parallelization, and tailored pretreatment techniques.

All computational tests were performed utilizing Gurobi Optimizer version 12.0.1 under an academic Web License Service (WLS) license. The solution was run with the following non-default settings: MIPFocus = 1, Heuristics = 0.5, Presolve = 2, Cuts = 2, and Threads = 0 (which lets it use up to 32 logical processors automatically). The limitations for default feasibility and optimality were kept the same (MIPGap = 10^{-4}). All of the tests were done on a Windows 11 workstation with an Intel Core i9-14900HX CPU. These parameters make sure that the computational framework is clear and can be repeated.

GINI Optimization Results

The Gurobi Optimizer is used to implement and solve the optimization model. It uses a branch-and-bound framework to solve mixed-integer linear programming problems. Gurobi's presolve routines, cutting-plane strategies, and ability to explore

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nodes in parallel make it possible to quickly find globally optimal solutions. To make sure that the results are clear and can be repeated, solver performance indicators like solution time, number of explored nodes, and optimality gap are reported.

In order to identify the performance of the results of optimization, one of the corridors of operation was selected for the optimization of the pertinent Gini index value. This corridor is served by a total number of 25 buses. The process employed for the identification of the 25 buses operating on the selected route is presented first in the following section, and the results of the optimization of the Gini index for the selected subset are thoroughly analyzed.

Selection of Buses for Optimization

In the process of choosing the 25 buses for incorporation into the optimization model, the complete fleet of 160 buses was initially ranked, in ascending order, by their realized mileage amounts. As 25 buses were required from the population, the division of 160/25, or 6.4, was taken as the base interval. Employing the method of Systematic Sampling, every 6th or 7th bus within the ranked list was selected and added into the sample set, thus ensuring the 25 chosen vehicles were a proper representation of the total fleet for mileage distribution purposes.

Furthermore, for the purpose of facilitating an appropriate comparison of the results of optimization, it proved desirable to recompute not just the hitherto calculated fleet-wide Gini index but also the resulting daily Gini index of 756 for the sampled size of 25 buses, thus forming the basis of both statistical as well as of trend analyses. This provides for an evenly comparable analytical baseline and assures that the improvements thus effected through the process of optimization can be aptly contextualized and validated through the same order of operations.

Table 4 presents the calculation of the areas under the Lorenz curve and between the Lorenz curve and the line of absolute equality for the 99th day, which corresponds to the day of highest heterogeneity in mileage distribution among the selected 25 buses. Accordingly, the Gini index for the 99th day—the day with the highest inequality—was calculated as 0.3956. This value is very close to the Gini index of the entire fleet of 160 buses (0.3814).

Total Area Under Equal Line	Total Area Under Lorenz Curve (B)	Area Between Curve and Line (A)	GINI (A/(A+B))
0.5	0.302212827	0.197787173	0.395574346

Table 4. GINI Index Calculation for Selected 25 Articulated Buses

Figure 4 illustrates the Lorenz curve corresponding to the Gini index on the 99th day for the same subset of 25 buses. Similarly, the estimated Lorenz curve for this sample subset exhibits a shape closely resembling that of the entire fleet.

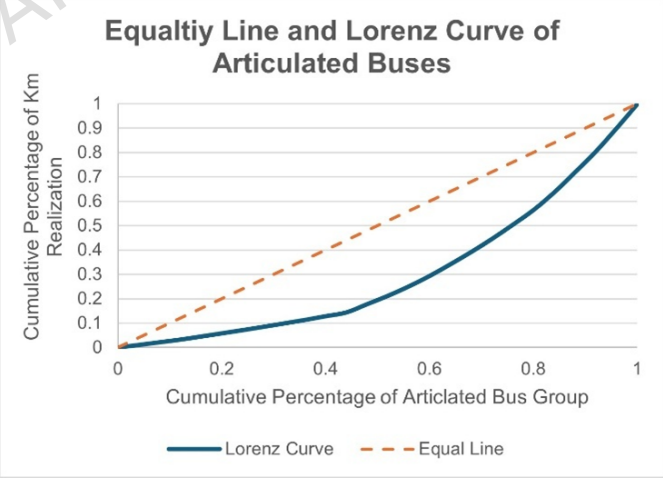


Figure 4. Equality Line and Lorenz Curve for Selected 25 Articulated Buses

Figure 5 illustrates the daily Gini index values for the selected 25 buses. An examination of this graph reveals a pattern highly similar to that of the entire fleet. Therefore, the interpretations and conclusions made in the previous subsection for the full fleet are equally applicable in this context.

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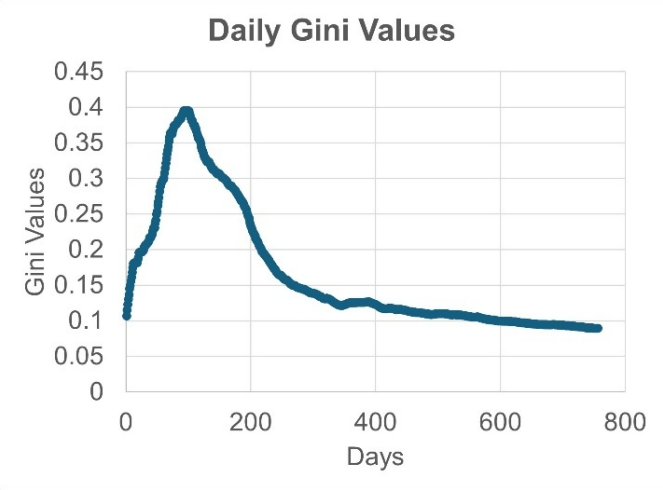


Figure 5. Daily Gini Index Values Graph for Selected 25 Articulated Buses

Baseline Allocation Strategies for Comparative Evaluation

To make the suggested optimization framework more reliable, two basic baseline allocation strategies were used for comparison:

- Random Assignment
- Simple Rotation Rule

The Random Assignment baseline randomly gives buses jobs that are possible while still meeting all operational requirements, such as full block coverage and the requirement that each bus only do one task. The technique was done 100 times because random allocation can lead to different results. We used the Gini index to find a sample benchmark distribution for each run and then derived descriptive statistics.

The Simple Rotation Rule baseline gives busses jobs in order of their ordered accumulated mileage. Buses with less accumulated mileage are given higher-mileage assignments first, which keeps the workload balanced without having to solve an optimization model.

These two basic methods give clear and low-computation benchmarks that can be used to judge how well the suggested Gini-based MILP model works.

Optimization Results

The proposed mixed-integer programming formulation was solved for a sampled route using the Gurobi Optimizer coupled with Python. The experiment considered 25 articulated buses, with the aim of generating a one-year assignment schedule that minimizes mileage inequality and improves annual mileage homogeneity.

Gurobi reported an initial problem size of 701 constraints (rows), 2,650 decision variables (columns), and 14,914 non-zero coefficients. Among the decision variables, 2,325 were integer, and after presolve the model was transformed to 745 rows and 2,274 columns with 24,461 non-zeros, including 1,934 binary variables. The root relaxation yielded an objective value of 2.268176×10^6 after 2,642 simplex iterations.

The branch-and-bound search explored 4,247 nodes and performed 438,697 simplex iterations, converging in 18.83 seconds while using up to 32 threads. The solver returned an optimal solution with a very small remaining optimality gap (0.0038%), where the best objective value was 6.422564612×10^6 and the best bound was 6.422321157×10^6. Under this optimized schedule, the resulting Gini index decreased to 0.0462, indicating a substantial improvement in mileage homogeneity compared with the historical/heuristic-based allocation for the same sub-fleet.

To illustrate the extent of enhancement, the optimized solution was juxtaposed with the two baseline allocation algorithms. For the Random Assignment baseline, 100 viable allocations were produced. The Gini values displayed a mean of 0.0616, a minimum of 0.0600, and a maximum of 0.0627 where the Simple Rotation Rule yielded a Gini coefficient of 0.06178. The histogram of Gini values for the Random Assignnment has been provided in Figure 6. The limited range signifies consistent levels of inequality within a random feasible allocation.

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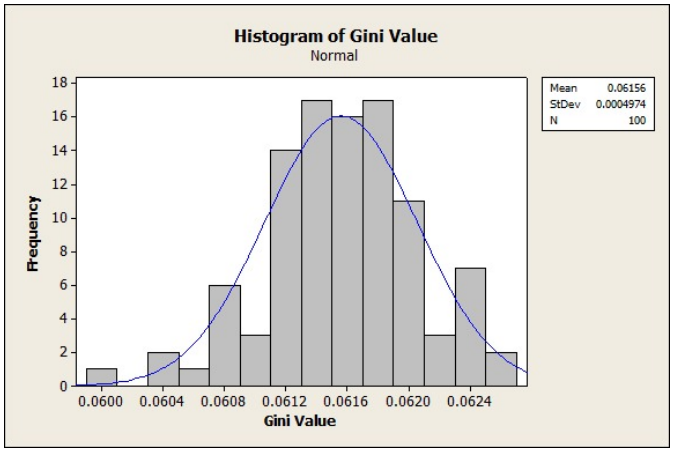


Figure 6. Daily Gini Index Values Graph for Selected 25 Articulated Buses

In comparison to these baselines, the proposed optimization model attained a significantly reduced Gini index of 0.0462. This signifies an estimated reduction of 25% compared to the random and rotation-based allocations, indicating that the noted enhancements result from the structured optimization framework rather than random fluctuations or basic workload ordering heuristics.

Moreover, the optimized solution consistently outperforms the entire distribution of random baseline results, as its Gini value (0.0462) lies well below the minimum observed random allocation value (0.0600), further confirming that the improvement is structurally driven by the optimization model. Table 5 demonstrates that the proposed MILP model achieves a substantially lower Gini coefficient compared to both random allocation and rotation-based baseline strategies.

Method	Mean Gini	Min Gini	Max Gini
Random (100 runs)	0.0616	0.0600	0.0627
Rotation	0.0618	–	–
MILP (Proposed)	**0.0462**	–	–

Table 5. Comparison of Gini Coefficient Results Across Allocation Methods

The Gini coefficient dropping from about 0.062 (baseline methodology) to 0.0462 means that the difference in effort between vehicles has become much smaller. Fleet managers can now reduce the number of cars that are significantly abused while still using certain cars that are not used as often. The decrease in mileage variability, as shown by the lower standard deviation, means that the maintenance plan is more dependable and in sync, the danger of early component degradation in some vehicles is lower, and long-term asset management is better. The Gini reduction we saw means that fleet work balance has improved both statistically and in terms of real-world operations.

The previous vehicle assignment schedule was compiled as an Excel spreadsheet, enabling detailed analysis and verification of day-by-day usage of blocks for a total of 365 operational days. The optimized schedule reveals considerable fairness improvement of vehicle usage, leading to a more balanced division of traveled kilometers.

Beyond the numerical advances, the results carry practical implications for fleet management via decreased differences inannual mileage accumulation, thereby facilitating optimized synchronicity of maintenance planning, decreased risk of over- and under-utilization, and increased lifecycles for vehicles. Enhanced homogeneity also facilitates sustainability of operational practices via fairness of resource utilization, toward the broader equity-motivated objectives for the public transit planning field.

These outcomes also reaffirm the suitability of the Gini index as an objective function for integer programming models. Compared with conventional cost-minimizing strategies, the model integrates fairness into the optimization process itself, presenting a balanced system that simultaneously accounts for efficiency and equity. To further evaluate the stability of the proposed framework under alternative assumptions, additional robustness analyses were conducted.

Robustness and Sensitivity Analysis

To further assess the stability of the proposed optimization framework, additional sensitivity analyses were conducted with respect to planning horizon assumptions and extreme initial mileage values.

First, the planning horizon was cut down from 365 days to 300 and 180 days, but the starting mileage levels stayed the same. The optimal Gini coefficient went up from 0.0462 (365-day horizon) to 0.0506 (300-day horizon) and then to 0.0616

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(180-day horizon). These results show that larger planning horizons make the model better at making mileage distribution more even. As the planning horizon shortens, the relative impact of pre-existing cumulative mileage levels intensifies, constraining the extent of feasible equalization within a reduced timeframe. Still, the optimization methodology is structurally sound and works well across all evaluated horizons.

Second, sensitivity tests were done by changing the very high initial mileage values while keeping the 365-day planning horizon. When the lowest accumulated mileage was cut by 10% and the maximum was raised by 10%, the Gini coefficient came out to be 0.0498. With a stronger change (-20% for the lowest and +20% for the greatest initial mileage), the Gini coefficient went up to 0.0535. These findings illustrate that, even in scenarios of heightened starting disparity, the suggested MILP structure preserves significant workload homogeneity capability.

The sensitivity analysis shows that the model is structurally strong even when the planning horizons and beginning mileage distributions change a little. The results also indicate that mileage homogeneity is fundamentally a medium- to long-term planning goal, wherein adequate temporal scope improves the efficacy of equity-driven optimization.

Operational Efficiency Implications

The main goal of the proposed model is to reduce mileage inequality, however it is important to see if this goal of fairness influences how well the model works. Total service production is kept the same in this formulation since each allocation technique must cover all operational blocks exactly once. So, the overall annual mileage for the system stays the same for all three solutions: random, rotation, and optimized.

The Gini index becomes better not by driving less miles, but by spreading the work out across the vehicles. In other words, the optimization framework doesn't cut down on service or vehicle use; it just changes the assignments so that the total number of kilometers driven by all the buses is the same each year.

From an efficiency point of view, the average annual mileage per vehicle stays the same as the total operational demand divided by the number of vehicles in the fleet. The MILP method, on the other hand, greatly reduces the spread of mileage across vehicles. This shows that equity can be improved without hurting operational output.

So, the results imply that in this case, mileage homogeneity and operational efficiency are not goals that are at odds with each other. In fact, balancing the workloads of vehicles may improve long-term operational stability by allowing maintenance cycles to be coordinated and cutting down on extreme vehicle abuse.

For the selected subset, the total annual mileage remained constant at 1,731,557.08 km across all allocation methods, while the standard deviation of annual vehicle mileage decreased from 29,674.64 km (baseline) to 16,449.99 km (MILP solution).

Discussion

The results of the present work illustrate the merits of considering equity-oriented objective functions for the optimization of public transport systems. Involving the Gini index for mileage distribution among buses and minimizing its value through optimization, the work reveals that equity can be concretely and numerically operationalized within optimization models. In distinction from the classical models where the main focus is on minimizing the cost or increasing the efficiency of the service, the Gini-based model accounts for the fairness of the vehicle usage, such that the operational loads are more evenly allocated throughout the fleet.

The comparison with baseline allocation schemes underscores the enhanced usefulness of incorporating the Gini index directly into the objective function. Both random allocation and basic rotation rules resulted in moderate workload balance (Gini $\approx$ 0.062), although neither method achieved the significantly lower inequality level attained by optimization (Gini = 0.0462). This suggests that structured equity-driven optimization offers quantifiable benefits over intuitive or heuristic balancing methods.

The optimization solutions demonstrate that for very complex fleet configurations, significant homogeneity improvements can be achieved within computationally manageable frames. Reduction of the Gini index to as low as 0.0462 reaffirms that differences in workload can be decreased substantially by careful block-to-vehicle task allocation strategies. The outcome, in turn, has critical managerial implications: through leveling annual mileage, the optimization not only increases fairness but also enables aligned maintenance scheduling, extends vehicle life, and reduces uncertainty of operations because of anomalous wear and tear.

Moreover, the proposed methodology is compatible with future equity-oriented models of transportation planning. Previous studies emphasized the hard task of balancing efficiency and fairness; however, the current analysis manifests operational efficiency and equity as not contradicting but complementing one another. In fact, mileage homogenization increases system sustainability by preventing the overuse of specific vehicles and underuse of others. Importantly, the reduction in mileage inequality does not come at the expense of operational efficiency, as total service output remains unchanged under the proposed framework. At the same time, the methodology provides an operational framework that can be enriched along other transit equity dimensions such as accessibility, frequency allocation, or environmental impact.

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The suggested framework can be put into place in existing transit agency workflows in a progressive way. The optimization model can be used at the yearly or semi-annual planning stage, when the current block schedules are already set. At this point, the model is a tool that helps make decisions by changing the vehicle-task combinations to make mileage more uniform without changing service coverage.

Second, the balanced annual mileage estimates can be sent directly to maintenance departments so that preventative maintenance cycles can be coordinated and the chances of parts wearing out too soon can be lowered. The framework doesn't replace existing operational planning; it only adds to it because the total service output stays the same.

Third, the model can be run again every month or every three months, for example, utilizing new cumulative mileage data from AVL or fleet management systems. This kind of rolling-horizon deployment would let agencies fix new imbalances slowly while still being able to change how they do things.

The suggested equity-oriented optimization methodology can be integrated into existing planning, assignment, and maintenance coordination processes, serving as an analytical improvement to extant decision-making routines.

In real-world transportation operations, the optimum schedule may not work out as planned if there are unanticipated problems, like a vehicle breaking down, being unavailable for a short time, or having to be moved to a different location. The suggested framework works best when operational conditions are steady over the planning horizon. However, the model structure is flexible enough to handle such changes through periodic re-optimization. In real-world situations, the optimization process can happen on a regular basis (like once a month or once a quarter), which lets new information about vehicle availability and mileage be used in the decision-making process. So, even if unforeseen operational shocks can momentarily throw off the balance of effort, the suggested equity-oriented framework can use adaptive re-planning to systematically restore homogeneity.

However, there are certain limitations to be considered. The model was tested for one route with 25 buses that, although representative enough, could not deliver the complete complexity of the entire operational network. In addition, the study utilized the day mileage data as the primary equity measure regardless of passenger demands and revenue emphases, both of which can arguably affect fairness observations. Future research can expand the scope by introducing multi-objective formulations where equity is paired with cost, quality of service, and environmental concerns and therefore having a more complete perspective.

The optimization was deliberately showcased on a representative sample of 25 busses to furnish a transparent and computationally feasible proof of concept. The proposed MILP formulation is not intrinsically constrained to this scale. The quantity of binary decision variables escalates in direct correlation with the number of buses and viable vehicle duties, while the count of auxiliary variables for Gini linearization expands quadratically with fleet size. Despite the rise in computational complexity, contemporary MILP solvers like Gurobi can manage far bigger problem instances, especially when utilizing presolve reductions and parallel processing. Consequently, the framework can be expanded to accommodate larger fleets or multi-route setups, contingent upon the availability of suitable computational resources.

From a methodological perspective, this study differs from prior equity-focused transportation research by embedding the Gini index directly into the optimization process rather than using it solely as an evaluative metric. Utilizing the Gini index as an objective function redirects the analytical emphasis from minimizing absolute mileage disparities to minimizing proportionate disparity among vehicles. This distinction is operationally significant, as it prevents scenarios in which a limited number of vehicles consistently bear disproportionately large workloads, despite an overall moderate dispersion. By including relative inequality into the decision-making process, the framework ensures that fairness is fundamentally integrated into fleet planning rather than considered a secondary concern. While existing studies predominantly assess inequality ex post, the proposed framework integrates equity into decision-making, enabling the systematic optimization of mileage homogeneity under realistic operational constraints. Although the present study adopts a vehicle-level perspective on equity, this operational viewpoint is closely connected to broader service- and user-level equity considerations. More homogeneous mileage allocation contributes to balanced vehicle condition, more predictable maintenance cycles, and reduced risk of unexpected service disruptions. In turn, this operational stability supports service reliability, which is a critical component of user-level equity. While the proposed framework does not directly optimize passenger accessibility or demographic fairness, it establishes an operational foundation upon which higher-level equity objectives can be systematically integrated in future multi-objective formulations.

The robustness analysis further indicates that the proposed framework maintains stable performance under alternative planning horizons and variations in initial mileage distributions, reinforcing the reliability of the model structure.

The findings of this research improve upon prior equity-focused transportation evaluations in several important ways compared to existing studies. While previous studies predominantly employed equity measurements such as the Gini index, Theil index, or accessibility measures as descriptive or evaluative tools, the present study incorporates equity directly into the optimization framework. The proposed paradigm, unlike earlier models that focused on cost minimization or efficiency maximization, emphasizes workload balance across vehicles under realistic operating constraints. This distinction enables a more prompt implementation of equality principles in routine operational choices, so augmenting and broadening the equity assessments outlined in previous transportation planning and sustainability studies.

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Conclusions

This study suggested an equity-focused optimization framework that directly includes the Gini index in the objective function of a bus fleet optimization model to make mileage more even under real-world operational constraints. The results show that minimizing mileage inequality can be systematically included in decision-making without making operations less feasible, using large-scale operational data from an urban bus network in Makkah. The optimized solutions show that the mileage spread between vehicles has clearly decreased, showing that treating equity as an operational goal rather than a post hoc evaluation metric is useful in practice. Sensitivity analyses conducted under varying planning horizons and modified initial mileage conditions further confirmed the structural stability of the proposed model.

From a planning and operational point of view, the results show that mileage homogeneity can help with both fairness between vehicles and bigger sustainability goals, like better use of assets and more predictable maintenance cycles. The study moves forward with public transportation planning methods that usually only focus on efficiency or cost minimization by putting equity into a mixed-integer optimization framework. So, the results give transportation authorities useful information that they can use to make sure that equity is taken into account when planning day-to-day operations.

The comparative assessment against random and rotation-based baselines demonstrates that the proposed approach significantly reduces workload disparity, underscoring its practical significance for transit fleet planning.

Even though the results are promising, this study has some problems that need to be pointed out.

First, the study only looks at one big city bus network, so the results may only apply to that network's specific operational characteristics. Second, the optimization framework only uses static historical mileage data and doesn't take into account changes in demand, real-time disruptions, or behavioral responses. Third, the Gini index is the only measure of equity. It works well for measuring mileage homogeneity, but it doesn't cover all aspects of transport equity, like accessibility, service reliability, or fairness at the user level.

There are a number of ways that future research can deal with these problems. To see if the proposed framework can be used in other cities or modes of transportation, it could be applied to multi-city or multi-modal transport systems. Adding dynamic demand patterns, real-time operational changes, or random elements could make the model even more realistic. In the future, studies may also look at policy-driven constraints or use more than one equity indicator along with the Gini index to better reflect social, spatial, and regulatory goals in public transport planning. Future studies may further investigate full-fleet implementations and explore decomposition-based or multi-stage optimization strategies to enhance scalability.

Data availability

The datasets generated and/or analysed during the current study are not publicly available due to confidentiality agreements with the Makkah public bus operator but are available from the corresponding author on reasonable request.

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The corresponding author, Burak Sevim, confirms that this declaration has been made on behalf of all authors.